Threshold Computation in the Head

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New Trends in PQC Workshop

Oxford, 11 June, 2024

CryptoExperts

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Threshold Computation in the Head

Joint work with Thibauld Feneuil

https://ia.cr/2022/1407

Original TCitH
framework
(Asiacrypt'23)

https://ia.cr/2023/1573

Improved TCitH
framework
(preprint)

Roadmap

- MPC-in-the-Head paradigm
- TC-in-the-Head framework (and application to PQ signatures)
    - 🌲 TCitH with Merkle trees
    - 🌲 TCitH with GGM trees
    - ✖ TCitH using multiplication homomorphism
    - 📦 TCitH using packed secret sharing
- Application: post-quantum ring signatures
- Relation to other proof systems

MPC-in-the-Head paradigm

MPC-in-the-Head paradigm

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

MPC-in-the-Head paradigm

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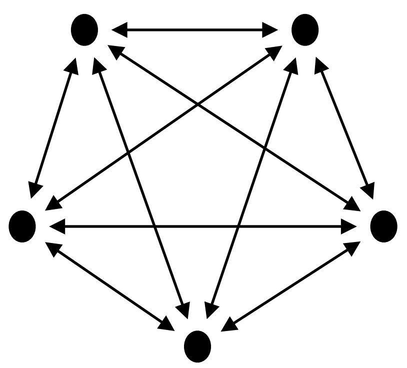
Multiparty computation (MPC)



Input sharing $[\![x]\!]$

Joint evaluation of:

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

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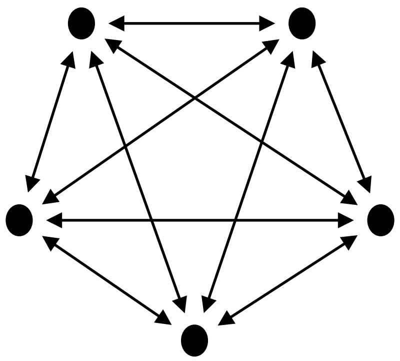

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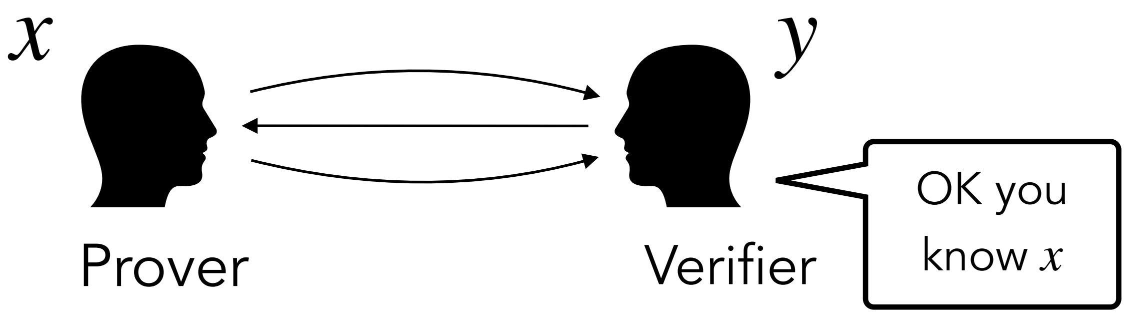
Zero-knowledge proof



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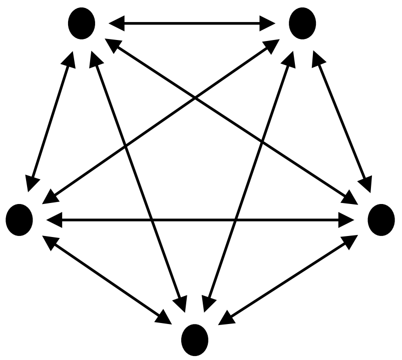

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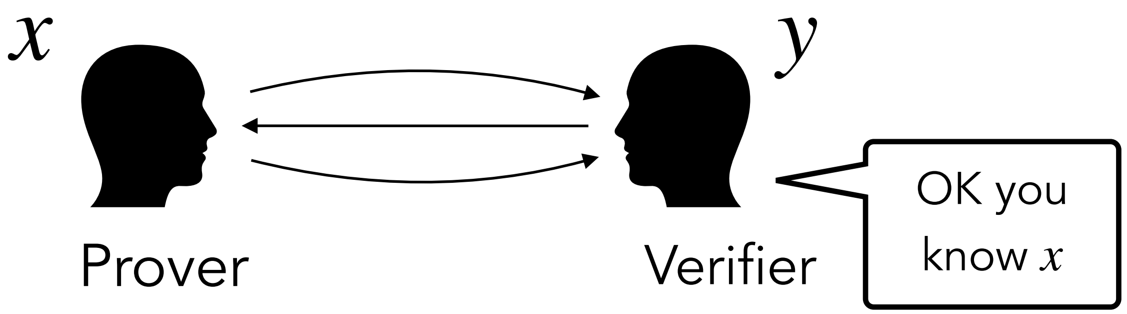
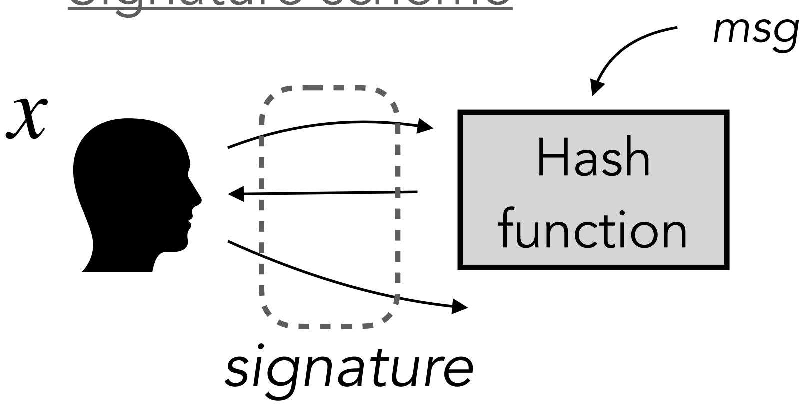


Signature scheme



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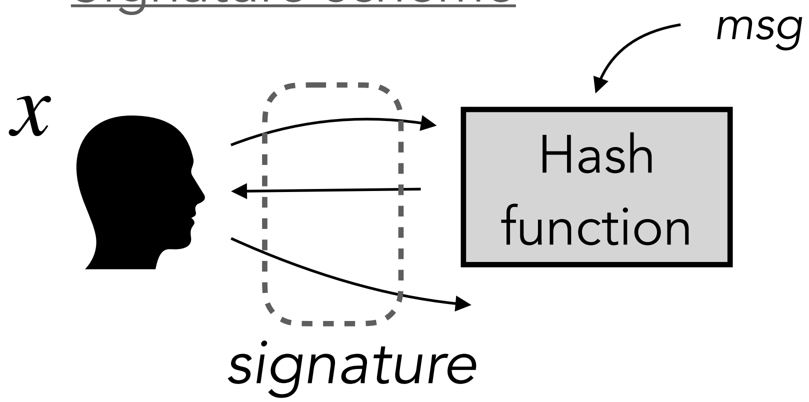
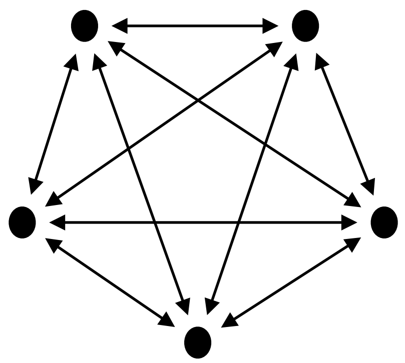


Multiparty computation (MPC)



Input sharing $[\![x]\!]$

Joint evaluation of:

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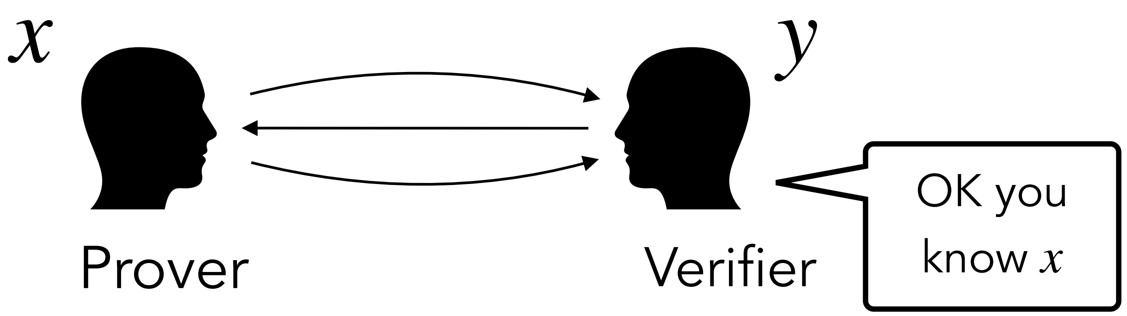
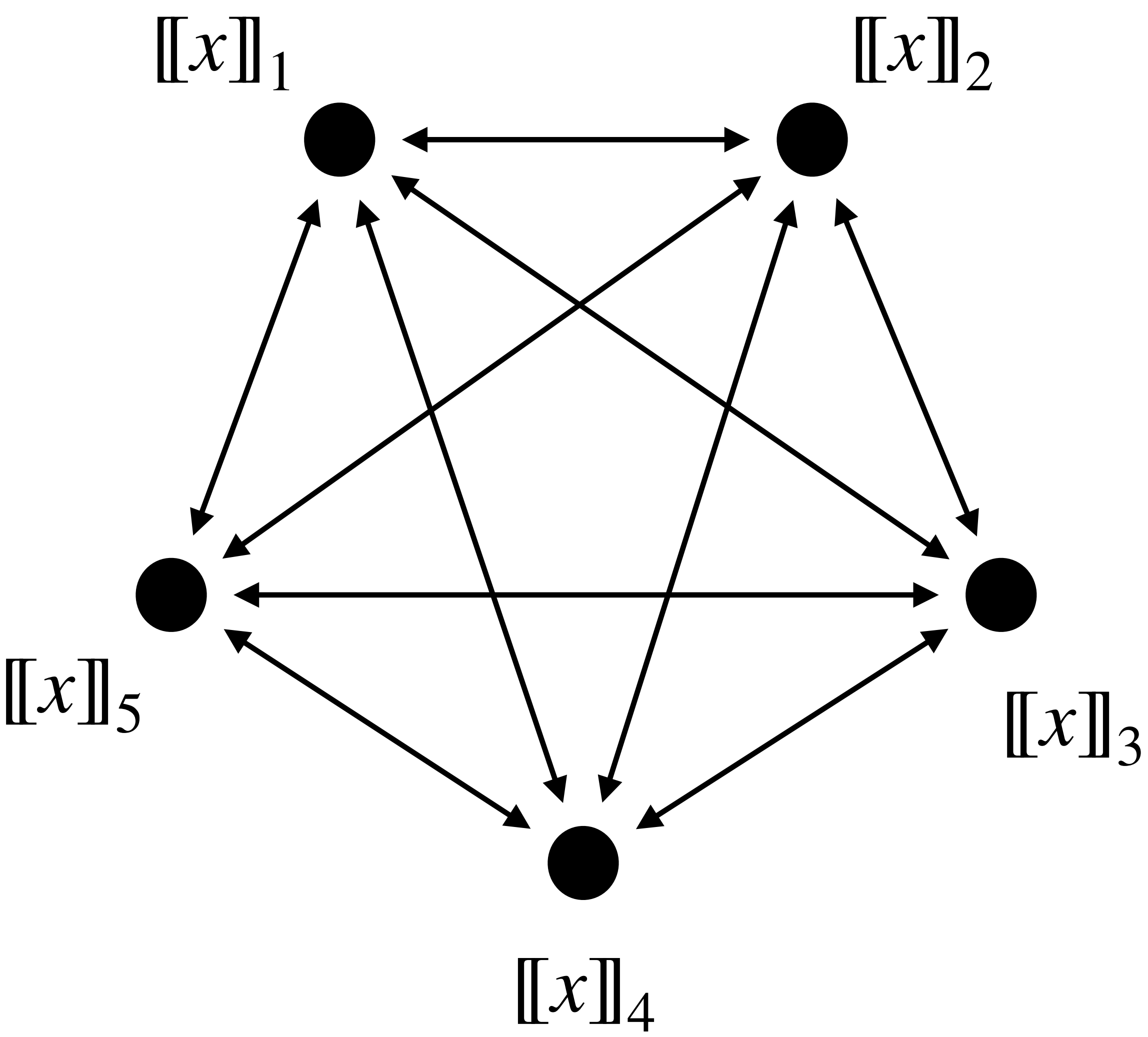
MPC-in-the-Head transform

Zero-knowledge proof



MPC model



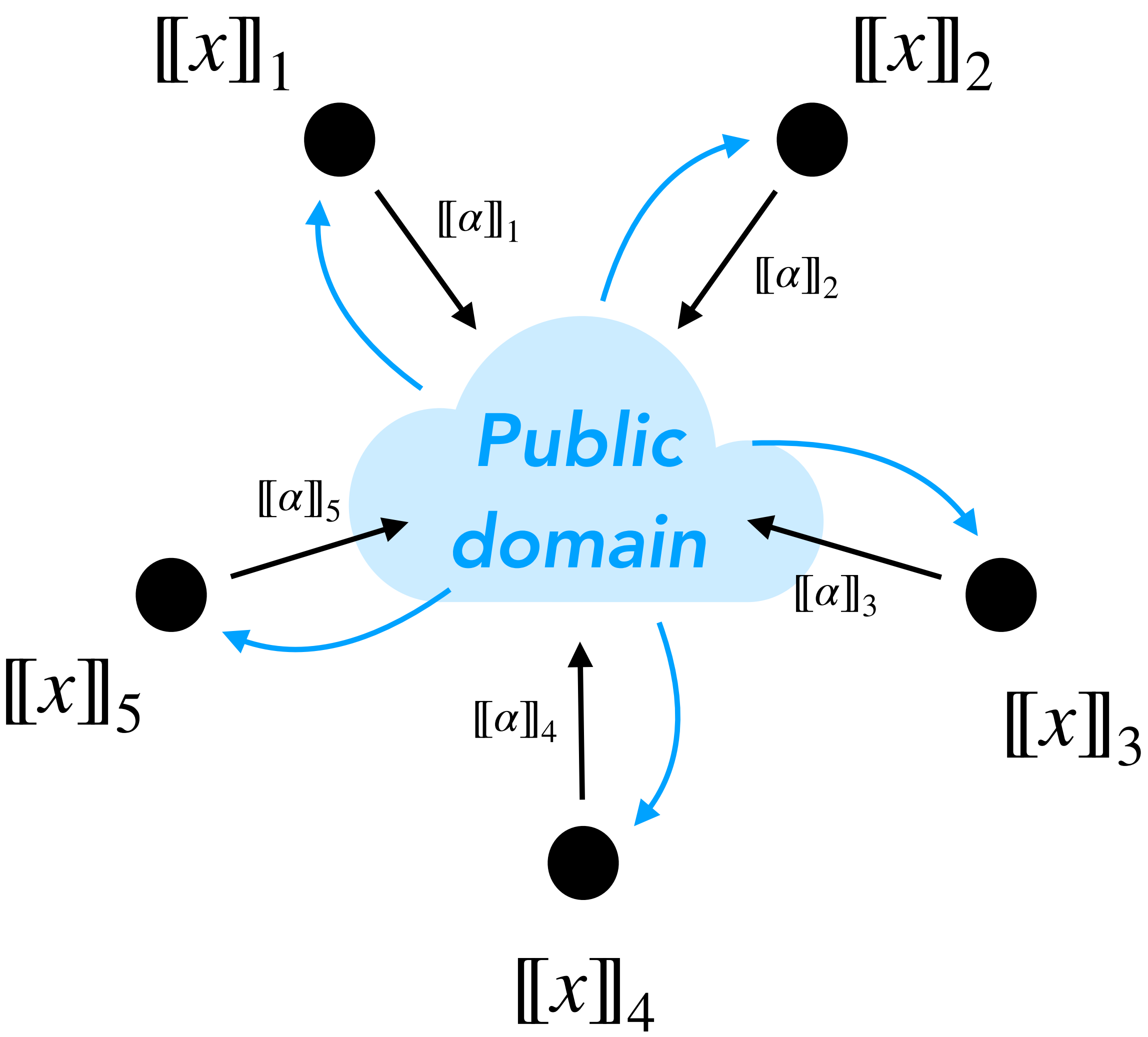
$[\![x]\!]$ is a linear secret sharing of x

- Jointly compute

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

- ℓ-private
- Semi-honest model

MPC model



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- ℓ-private
- Semi-honest model
- *Broadcast model*

MPCitH transform

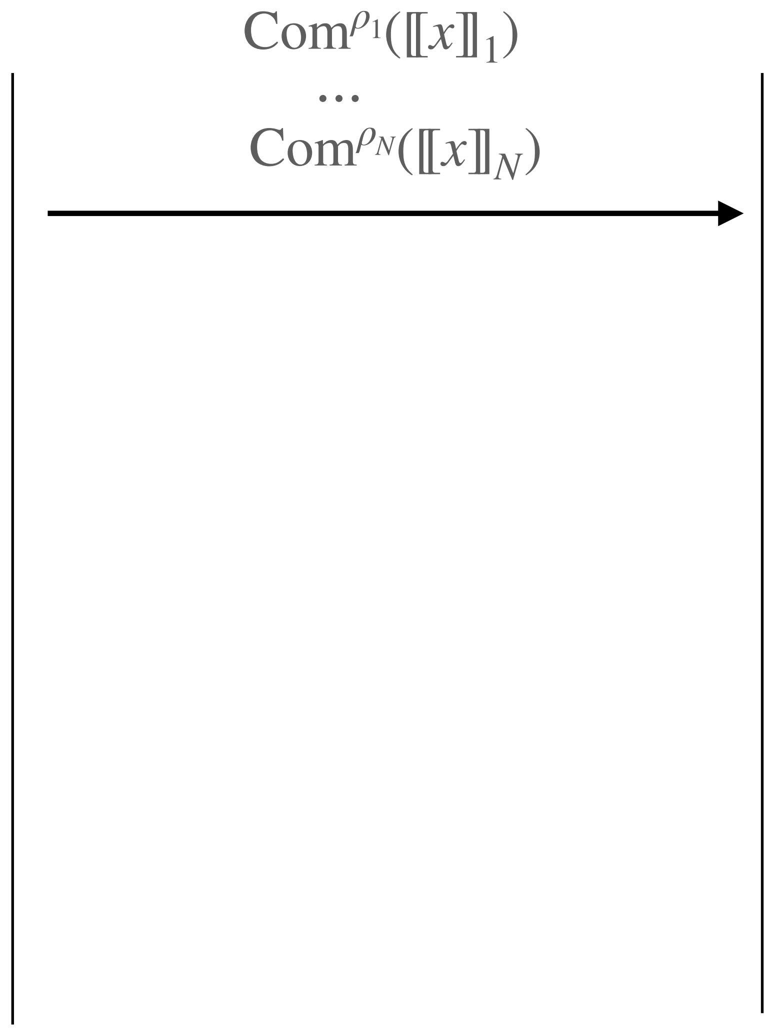
Prover

Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$



$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\cdots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

Prover

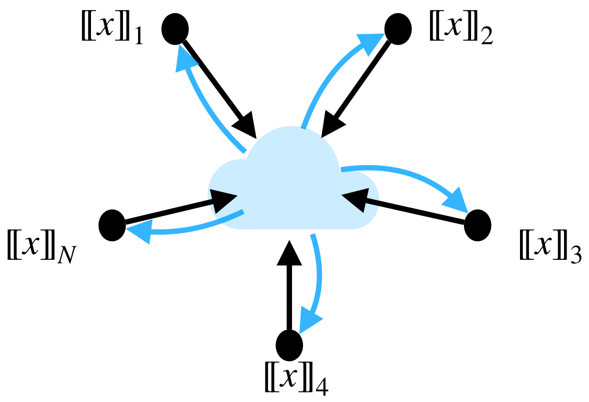
Verifier

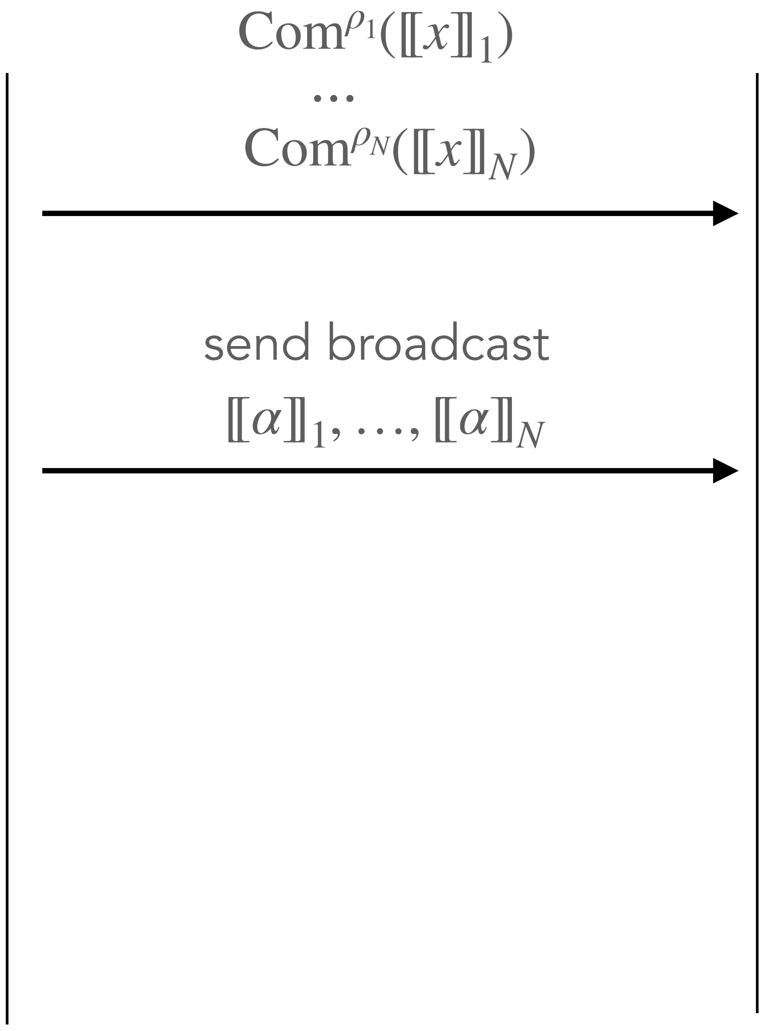
MPCitH transform

① Generate and commit shares

$$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$$

② Run MPC in their head





$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

…

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

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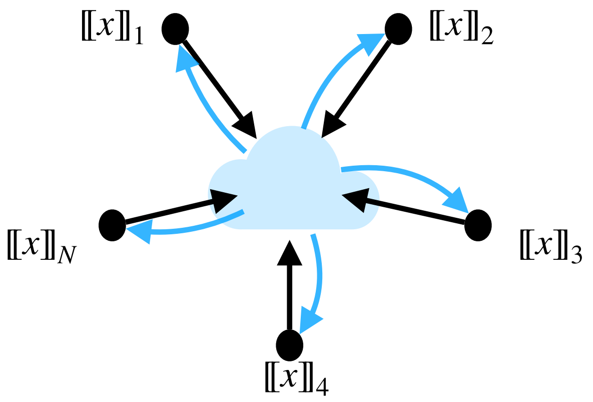

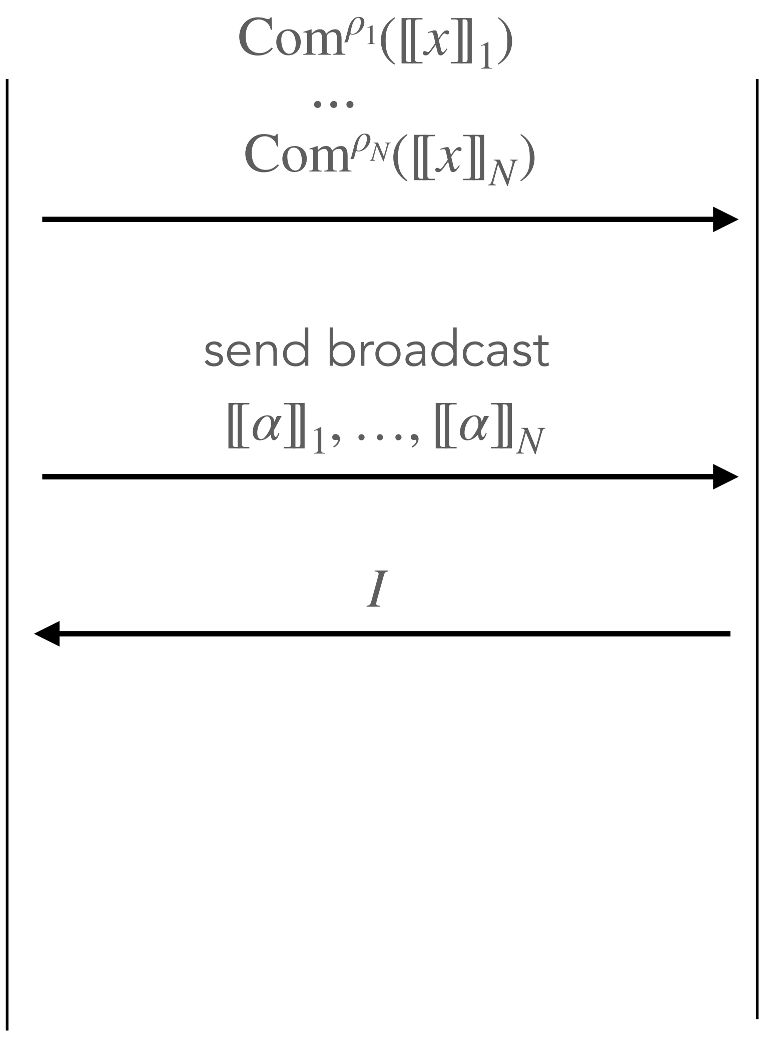

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send broadcast

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I

③ Choose a random set of parties

$I \subseteq \{1,\ldots,N\}$, s.t. $|I| = \ell$.

Prover

Verifier

MPCitH transform

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$\llbracket x \rrbracket = (\llbracket x \rrbracket_1, \ldots, \llbracket x \rrbracket_N)$

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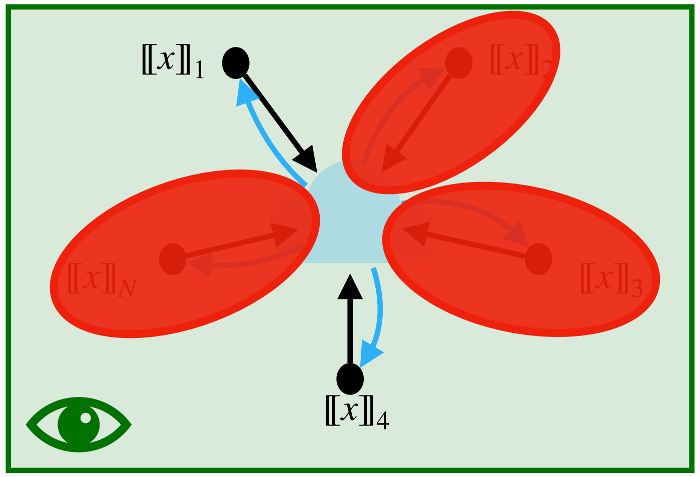
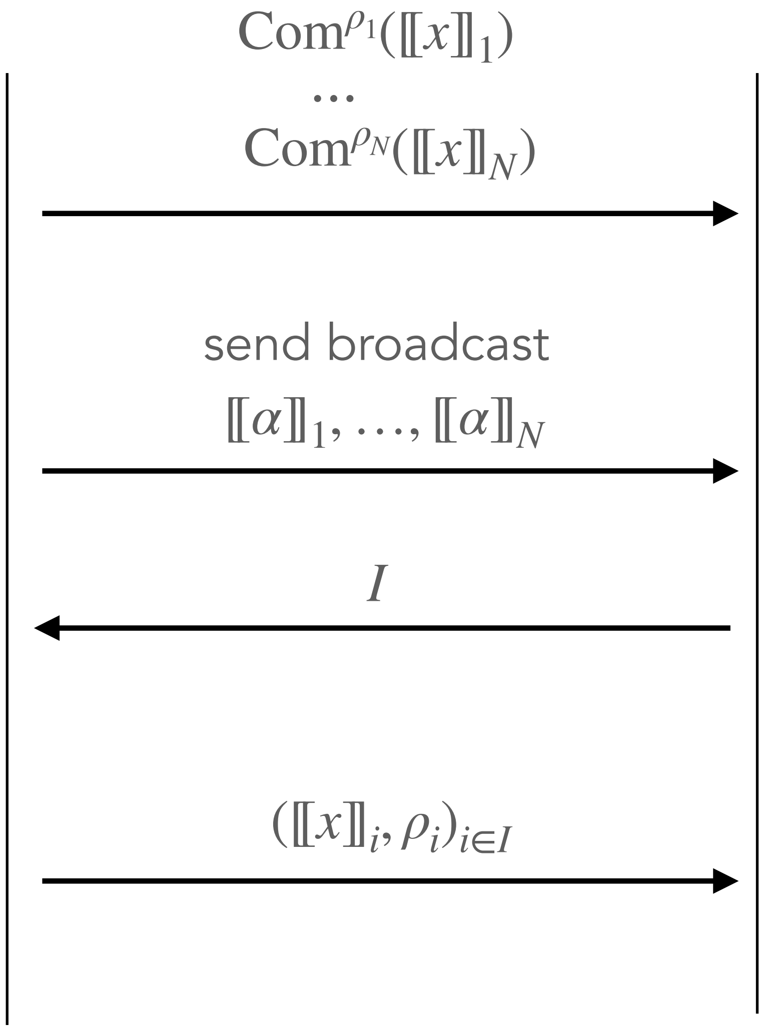


$\llbracket x \rrbracket_1$
$\llbracket x \rrbracket_2$
$\llbracket x \rrbracket_N$
$\llbracket x \rrbracket_3$
$\llbracket x \rrbracket_4$

④ Open parties in I



$\mathrm{Com}^{\rho_1}(\llbracket x \rrbracket_1)$

$\ldots$

$\mathrm{Com}^{\rho_N}(\llbracket x \rrbracket_N)$

send broadcast

$\llbracket \alpha \rrbracket_1, \ldots, \llbracket \alpha \rrbracket_N$

I

$(\llbracket x \rrbracket_i, \rho_i)_{i \in I}$

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Prover

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MPCitH transform

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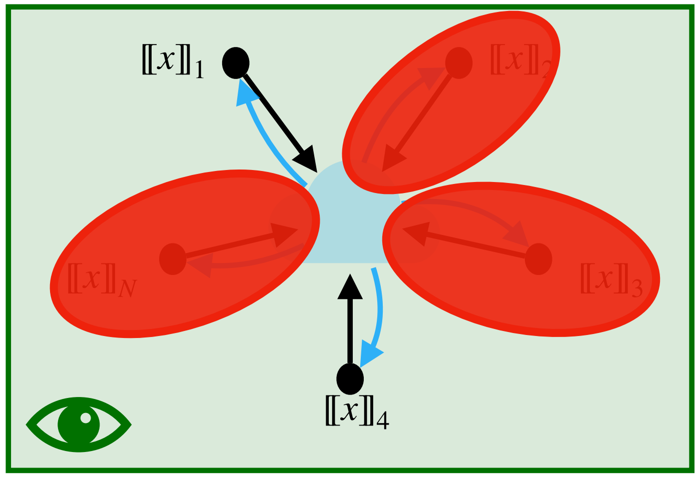

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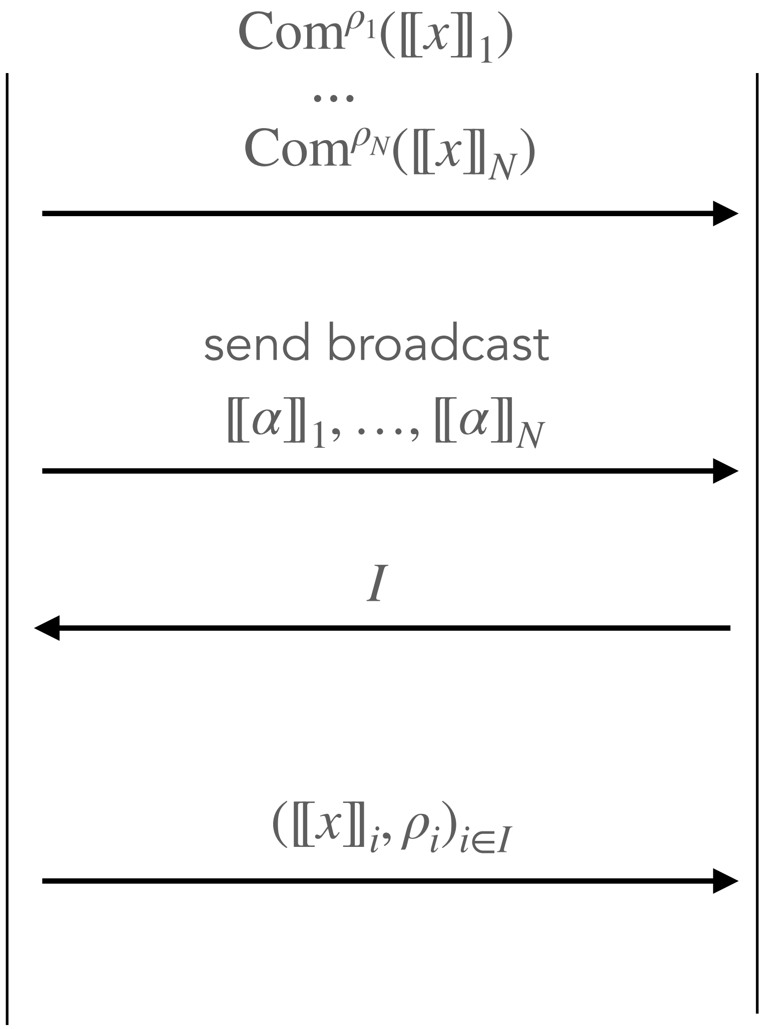

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③ Choose a random set of parties

$I \subseteq \{1, \ldots, N\}$, s.t. $|I| = \ell$.

⑤ Check $\forall i \in I$

- Commitments $\mathrm{Com}^{\rho_i}([\![x]\!]_i)$
- MPC computation $[\![\alpha]\!]_i = \varphi([\![x]\!]_i)$

Check $g(y, \alpha) =$ Accept

Prover

Verifier

MPCitH transform: with additive sharing

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$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

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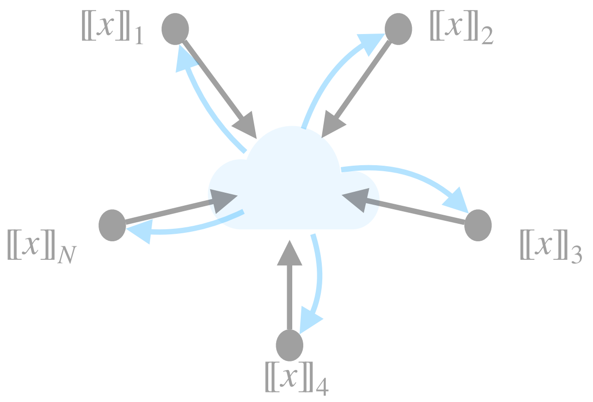

send broadcast

Additive sharing:

$$x = [\![x]\!]_1 + \cdots + [\![x]\!]_N$$

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④ Open parties in I

$([\![x]\!]_i, \rho_i)_{i \in I}$

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$\dots$

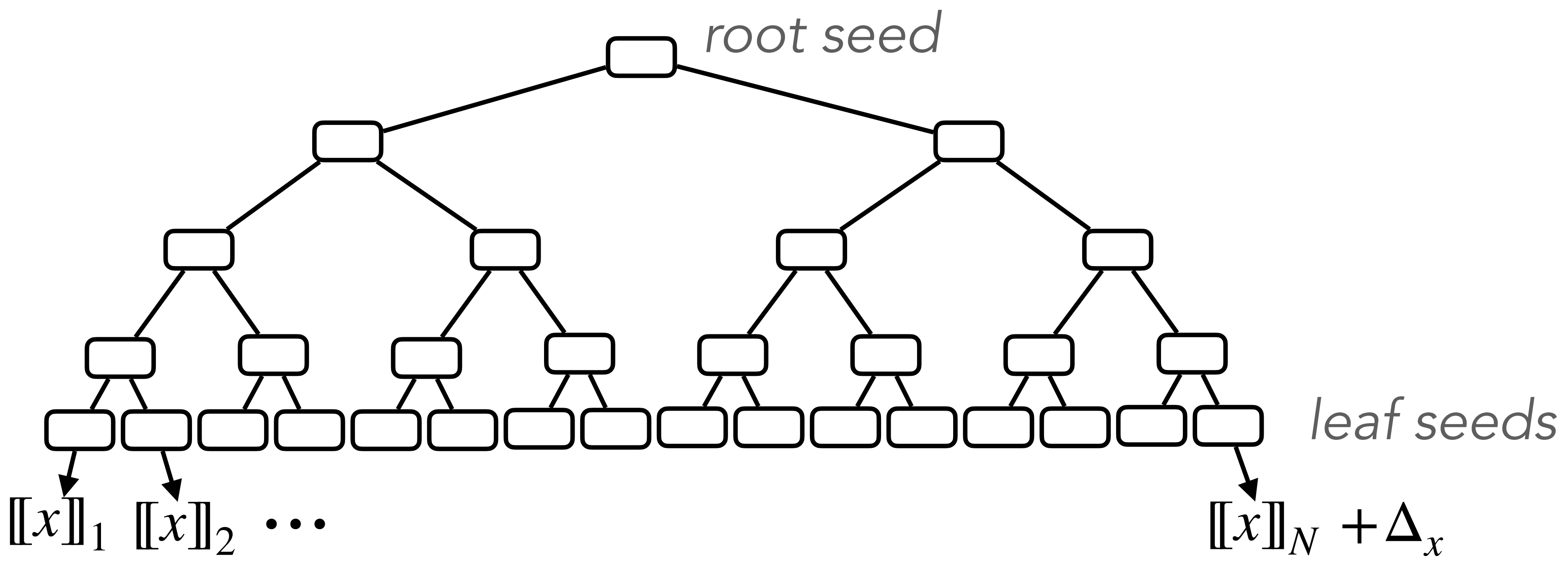
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

② Run MPC in their head

es

$([\![x]\!]_i)$

Generated using a GGM seed tree [KKW18]:



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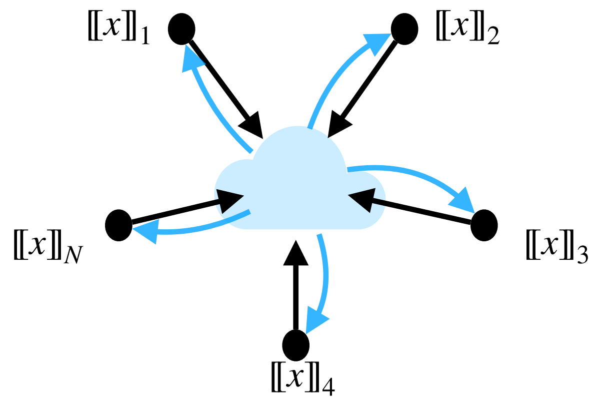

④ Open parties in I

Sharing / MPC protocol

$(N-1)$-*private*

parties

ℓ .

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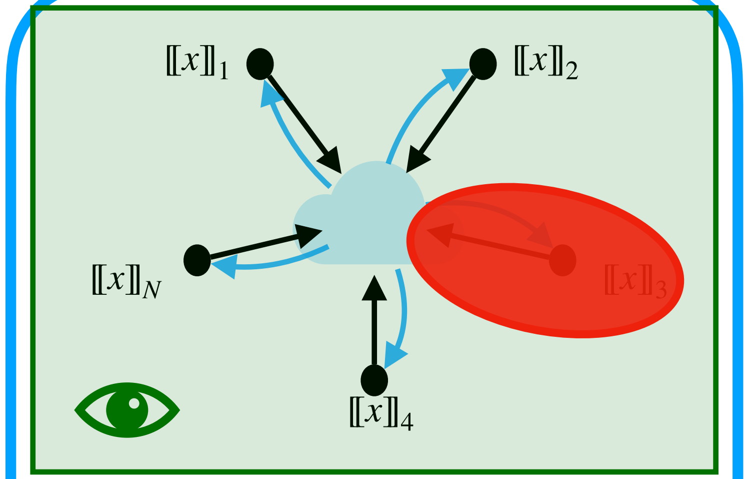

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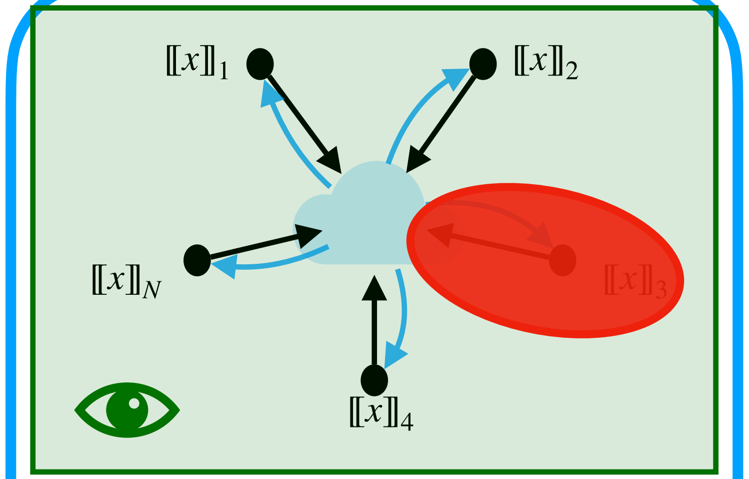

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$(N-1)$-*private*

$\Rightarrow$ soundness error = $\dfrac{1}{N}$

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$\cdots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

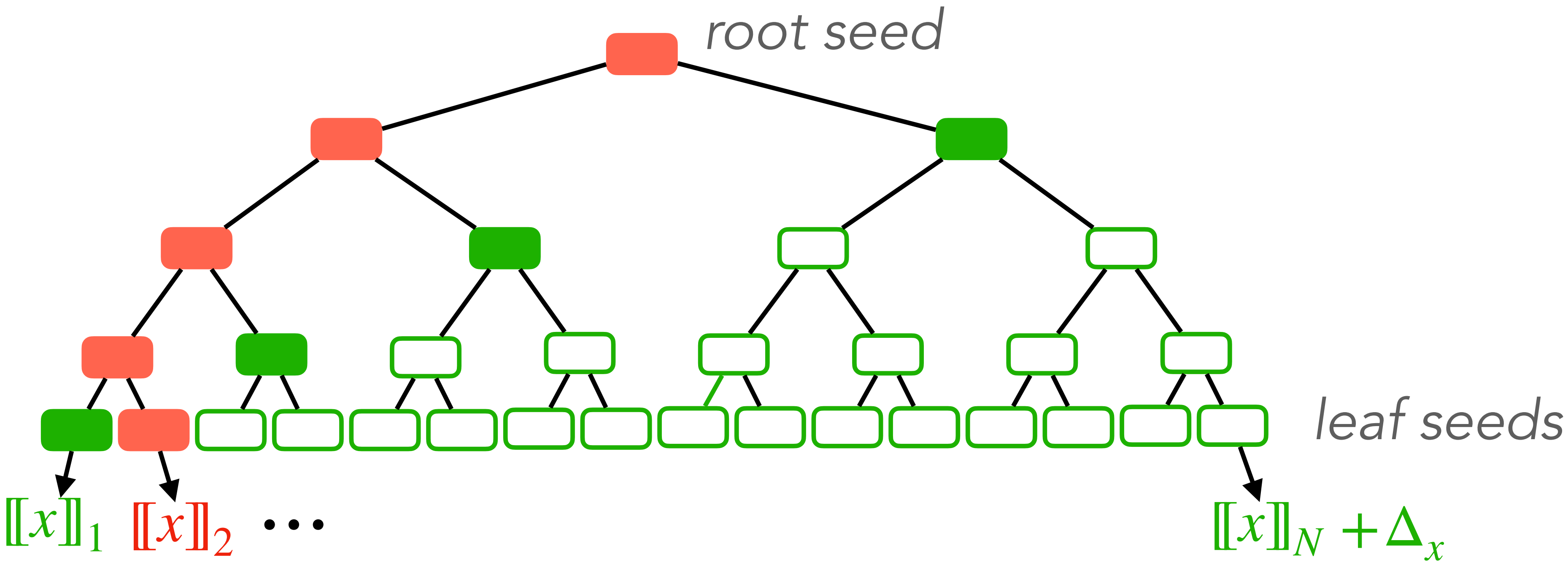
es

ties

$([\![x]\!]_i)$

$\varphi([\![x]\!]_i)$

Only $\log_2 N$ seeds to be revealed:



TC-in-the-Head framework (with Merkle trees)

Threshold Computation in the Head

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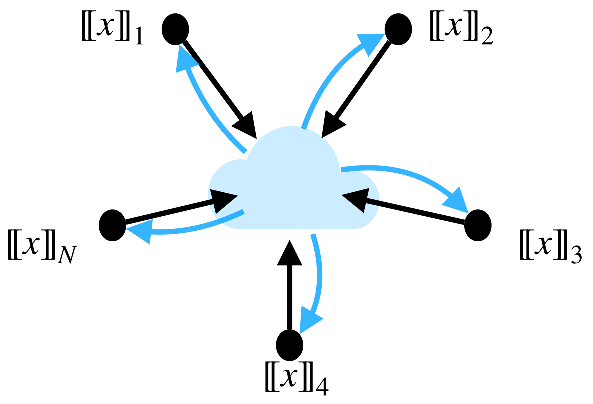
② Run MPC in their head



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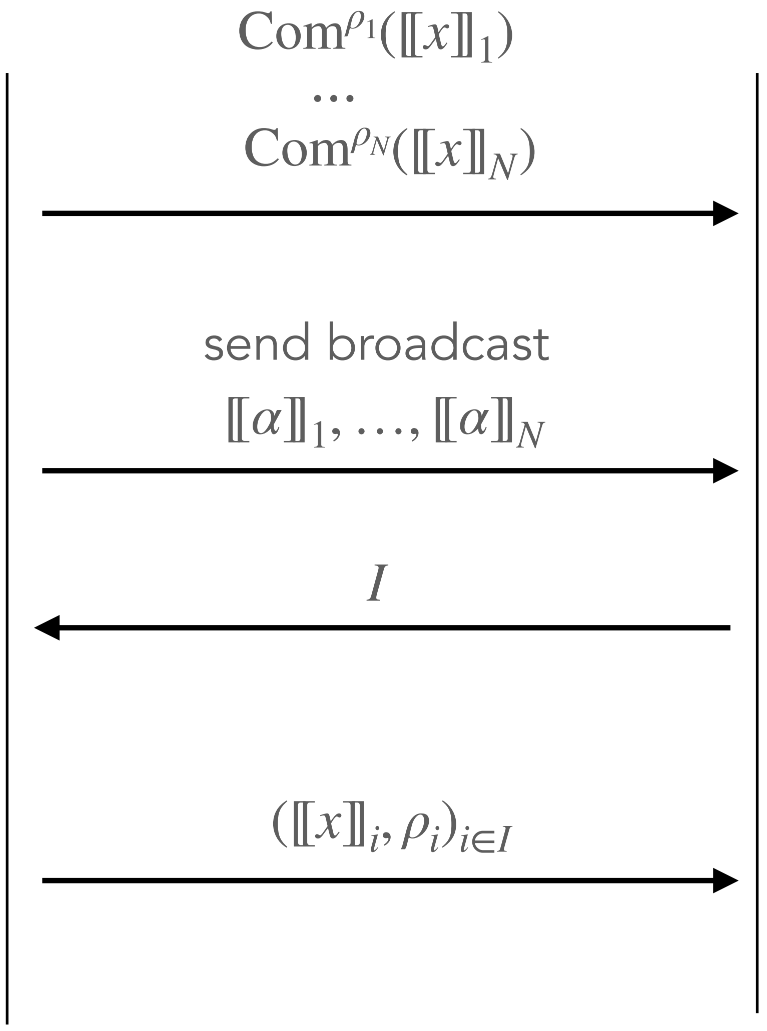

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Prover

Verifier

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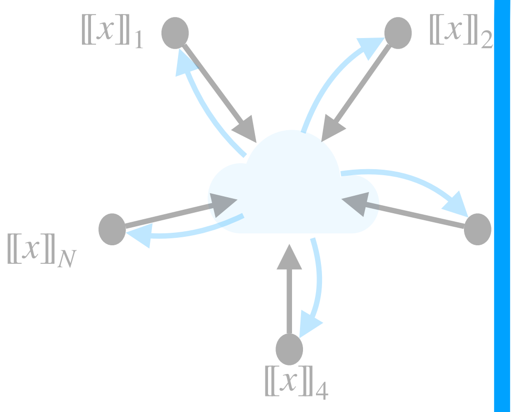
② Run MPC in their hea



Shamir secret sharing:

$$[\![x]\!]_i := P(e_i) \quad \forall i$$

$$\text{for} \;\; P(X) := x + r_1 \cdot X + \cdots + r_\ell \cdot X^\ell$$

n set of parties

t. $|I| = \ell$.

$\mathrm{Com}^{\rho_i}([\![x]\!]_i)$

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ccept

④ Open parties in I

Prover

Verifier

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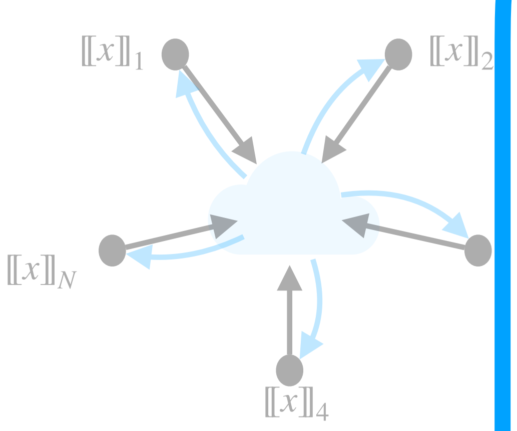

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$\Rightarrow \ell$-privacy

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Prover

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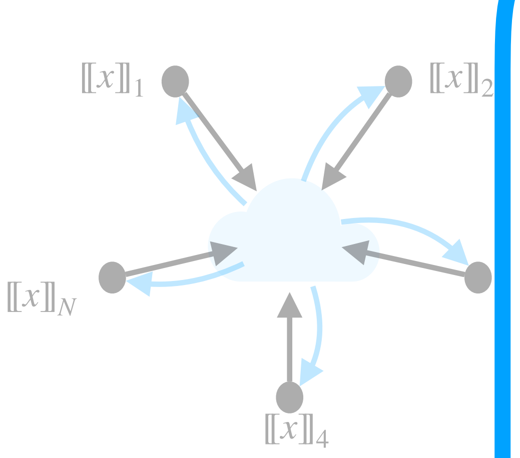

$\llbracket x \rrbracket_1$
$\llbracket x \rrbracket_2$
$\llbracket x \rrbracket_N$
$\llbracket x \rrbracket_4$

④ Open parties in I

m set of parties
t. $|I| = \ell$.

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ion $\llbracket \alpha \rrbracket_i = \varphi(\llbracket x \rrbracket_i)$
ccept

Shamir secret sharing:

$$\llbracket x \rrbracket_i := P(e_i) \quad \forall i$$

for $P(X) := x + r_1 \cdot X + \cdots + r_\ell \cdot X^\ell$

$\Rightarrow \ell$-privacy

We use $\ell \ll N$ (e.g. $\ell = 1$)

Prover

Verifier

Threshold Computation in the Head

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\ldots$

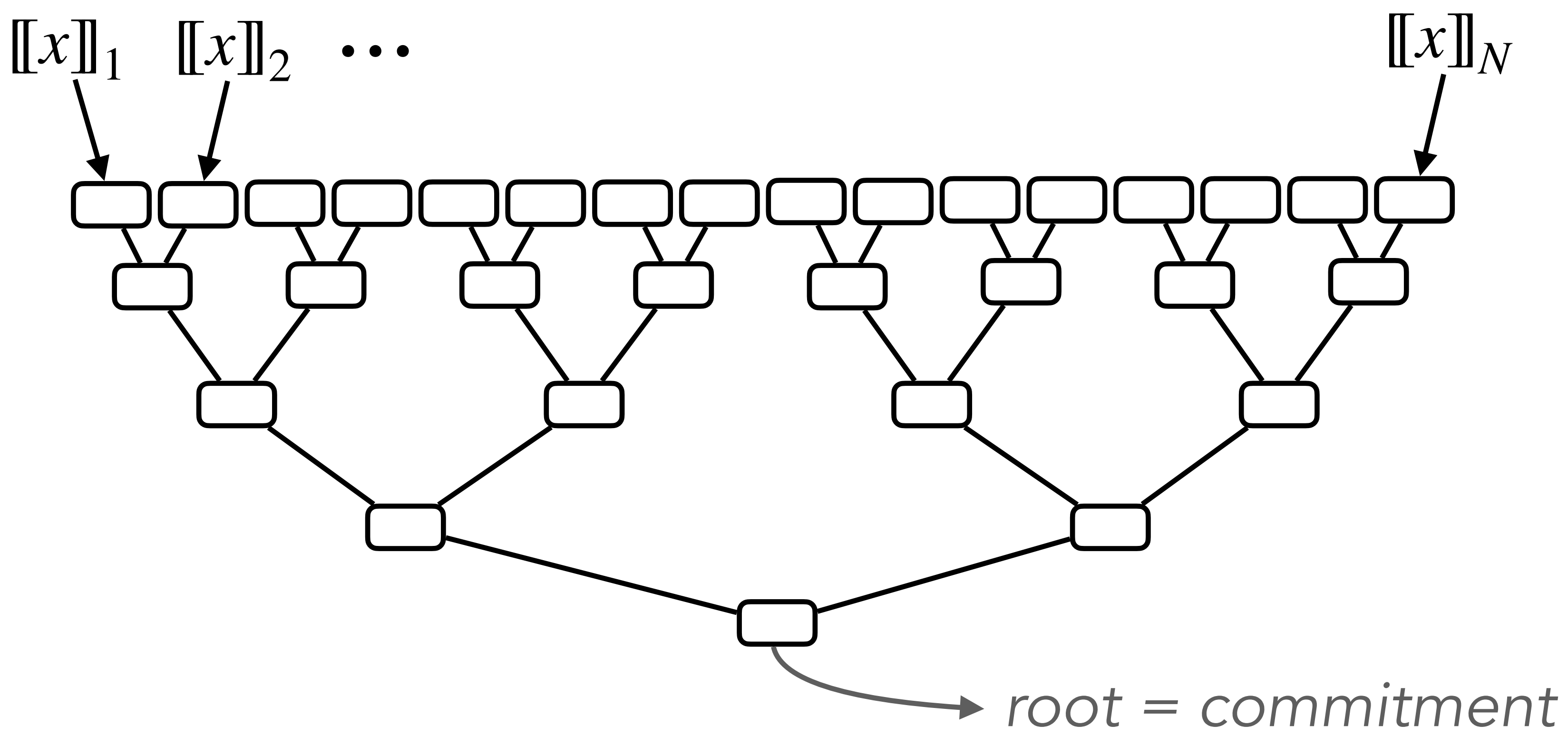
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

② Run MPC in their head

es

$([\![x]\!]_i)$

Committed using a Merkle tree:



Threshold Computation in the Head

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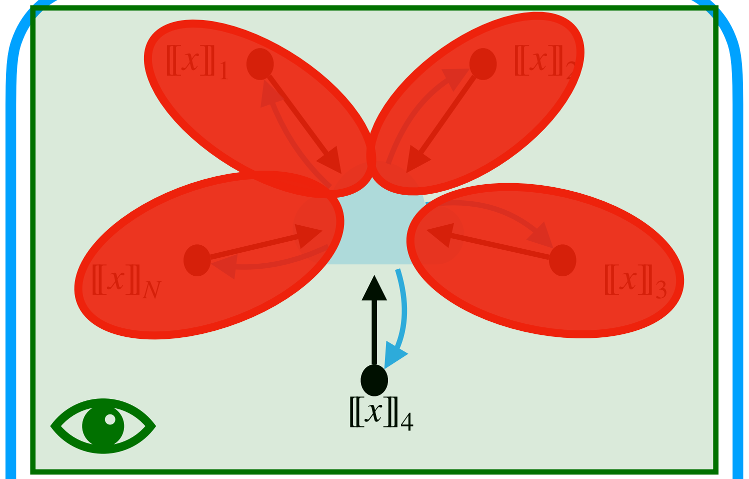
② Run MPC in their head



④ Open parties in I

Prover

Sharing / MPC protocol *ℓ-private*

parties

ℓ.

$]\!]_i)$

$= \varphi([\![x]\!]_i)$

Verifier

Threshold Computation in the Head

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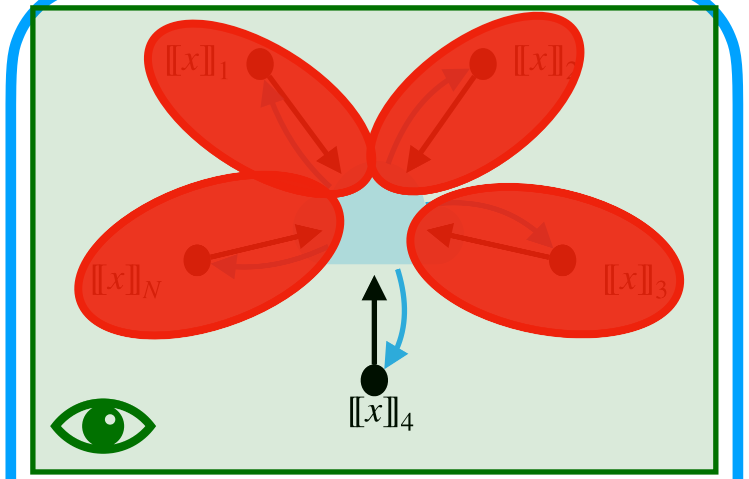
② Run MPC in their head



④ Open parties in I

Sharing / MPC protocol *ℓ-private*

⇒ soundness error = $(N - \ell)/N$ 🤔

parties

ℓ.

$]\!]_i)$

$= \varphi([\![x]\!]_i)$

Prover

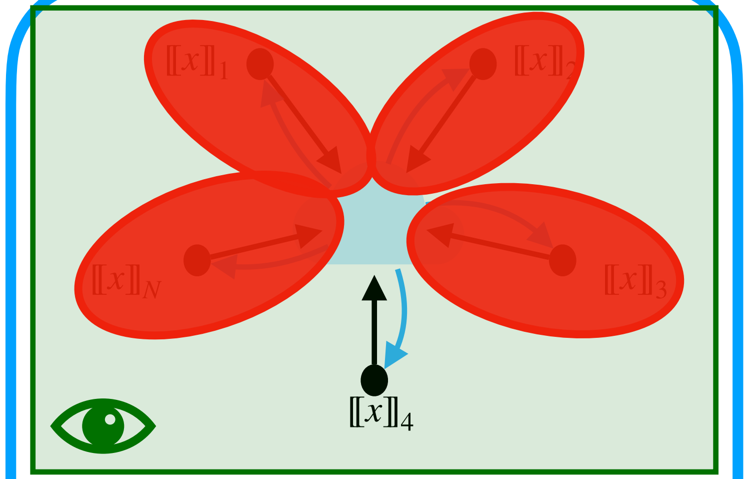
Verifier

Threshold Computation in the Head

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \dots, [\![x]\!]_N)$

② Run MPC in their head



④ Open parties in I

Prover

Sharing / MPC protocol *ℓ-private*

⇒ soundness error = $(N - \ell)/N$ 🤔

💡 broadcast messages must be valid Shamir's sharings

parties

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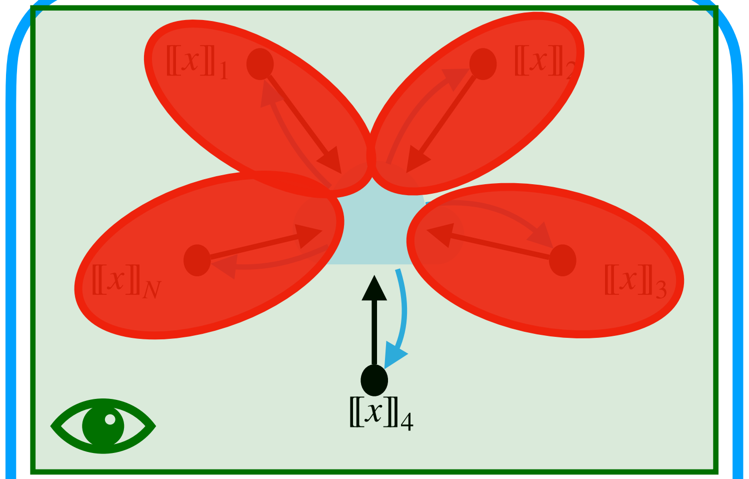

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⇒ soundness error = $\dfrac{1}{\binom{N}{\ell}}$ 🤩

parties

ℓ.

$]\!]_i)$

$= \varphi([\![x]\!]_i)$

Verifier

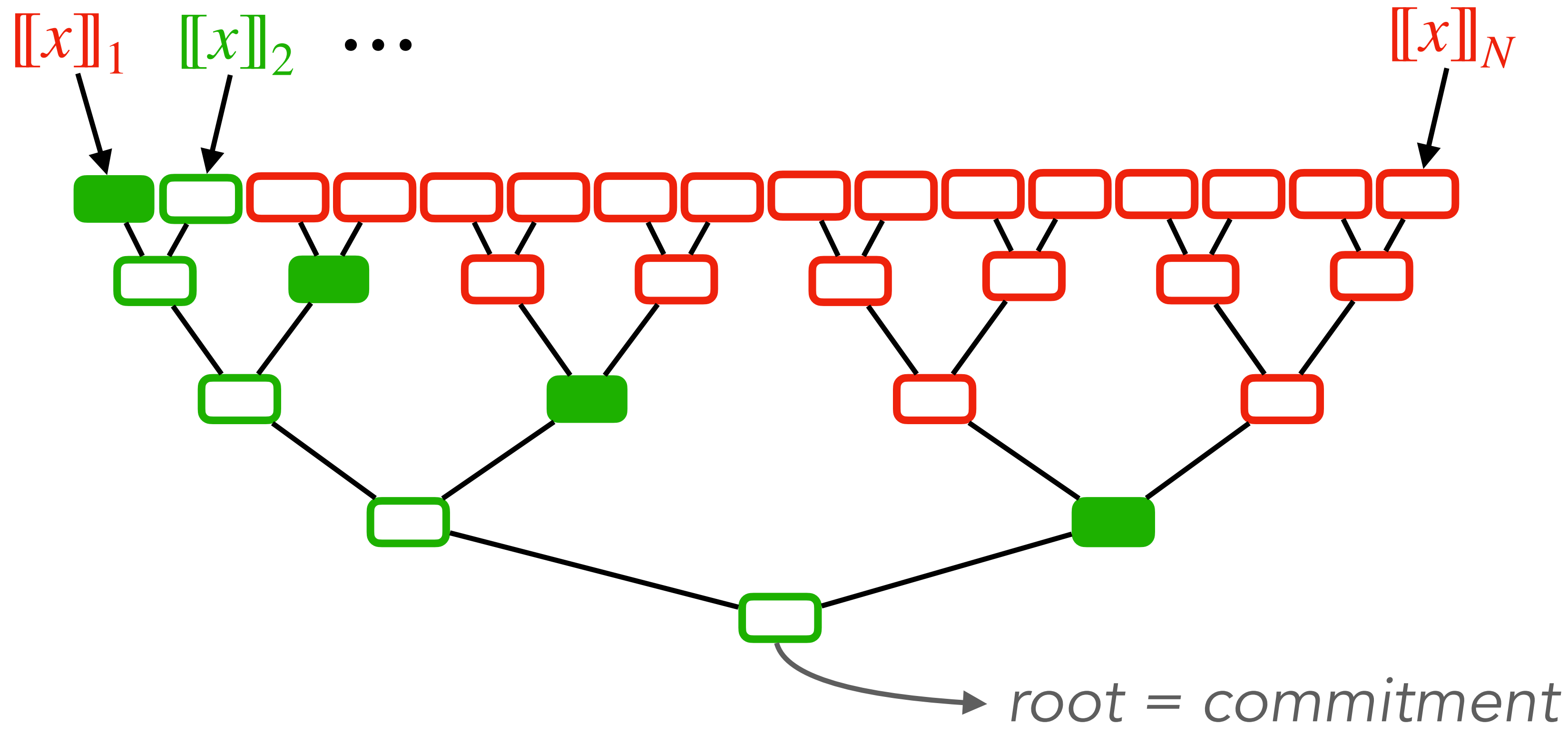
Threshold Computation in the Head

① Generate and commit shares $\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

ties

$\rho([\![x]\!]_i)$

Only $\log_2 N$ labels to be revealed:



Soundness

$$\begin{aligned} p &= \text{“false positive probability”} \\ &= P\left[\text{MPC protocol accepts } [\![x]\!] \text{ while } f(x) \neq y\right] \end{aligned}$$

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Soundness error of standard MPCitH

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$$\begin{tikzcd} \boxed{\dfrac{1}{N} + p} \arrow[r, bend left, "\text{hope 🙏}"] & \boxed{\dfrac{1}{\binom{N}{\ell}} + p} \end{tikzcd}$$

Soundness error of standard MPCitH

Soundness error of TCitH

Soundness

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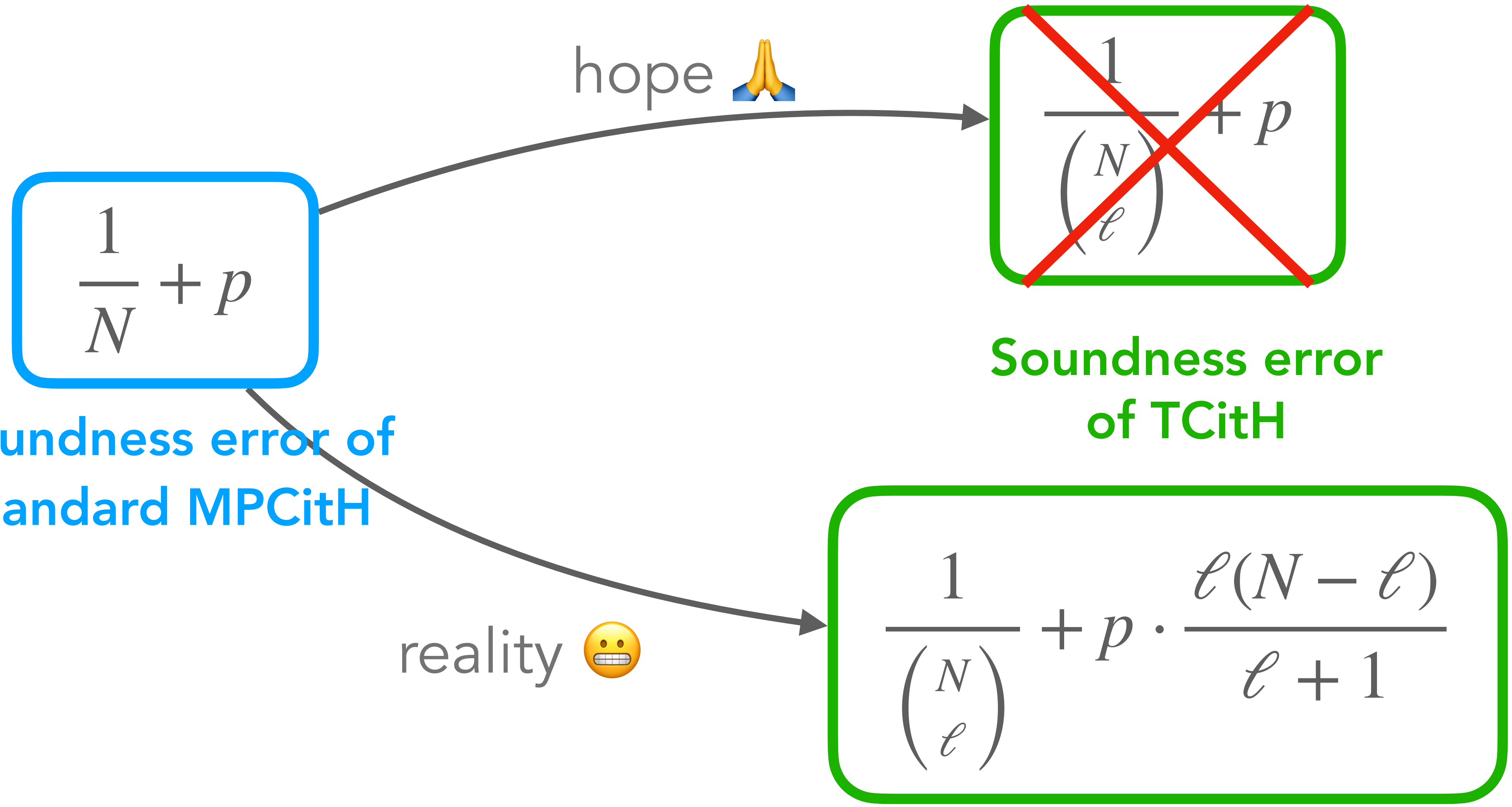
$$= P\left[\text{MPC protocol accepts } [\![x]\!] \text{ while } f(x) \neq y\right]$$



$$\frac{1}{N} + p$$

Soundness error of standard MPCitH

hope 🙏

$$\xcancel{\frac{1}{\binom{N}{\ell}} + p}$$

Soundness error of TCitH

reality 😬

$$\frac{1}{\binom{N}{\ell}} + p \cdot \frac{\ell(N-\ell)}{\ell+1}$$

Soundness

$$\frac{1}{\binom{N}{\ell}} + p \cdot \frac{\ell(N-\ell)}{\ell+1}$$

Why?

🤔

Soundness

$$\frac{1}{\binom{N}{\ell}} + p \cdot \boxed{\frac{\ell(N-\ell)}{\ell+1}}$$

Why?

- Prover can commit invalid sharings
- Let $[\![x]\!]^{(J)}$ = sharing interpolating $\left([\![x]\!]_i\right)_{i \in J}$
- Many different $[\![x]\!]^{(J)} \Rightarrow$ many possible false positives

Soundness

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Why? 🤔

💡

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🛠

- "Degree-enforcing commitment scheme"
- Verifier → Prover : random $\{\gamma_j\}$
- Prover → Verifier : $[\![\xi]\!] = \Sigma_j \, \gamma_i \cdot [\![x_j]\!]$
- Before MPC computation

Soundness

$$\frac{1}{\binom{N}{\ell}} + p \cdot \frac{\ell(N-\ell)}{\ell+1}$$

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$$\Longrightarrow \boxed{\frac{1}{\binom{N}{\ell}} + p}$$

TCitH vs. standard MPCitH

$\ell = 1 \;\Rightarrow\;$ Similar soundness: $\dfrac{1}{N} + p$ 🤝

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	MPCitH + seed trees + hypercube [AGHHJY23]	**TCitH** $\ell = 1$
Prover runtime	Party emulations: $\log N + 1$ Symmetric crypto: $O(N)$	Party emulations: 2 Symmetric crypto: $O(N)$

😀

fewer party emulations

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🚀 *much less symmetric crypto*

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Size of tree	128-bit security: ~2KB 256-bit security: ~8KB	128-bit security: ~4KB 256-bit security: ~16KB	🙁

×2

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Number of parties		$N \leq \|\mathbb{F}\|$	😀

Getting rid of these limitations

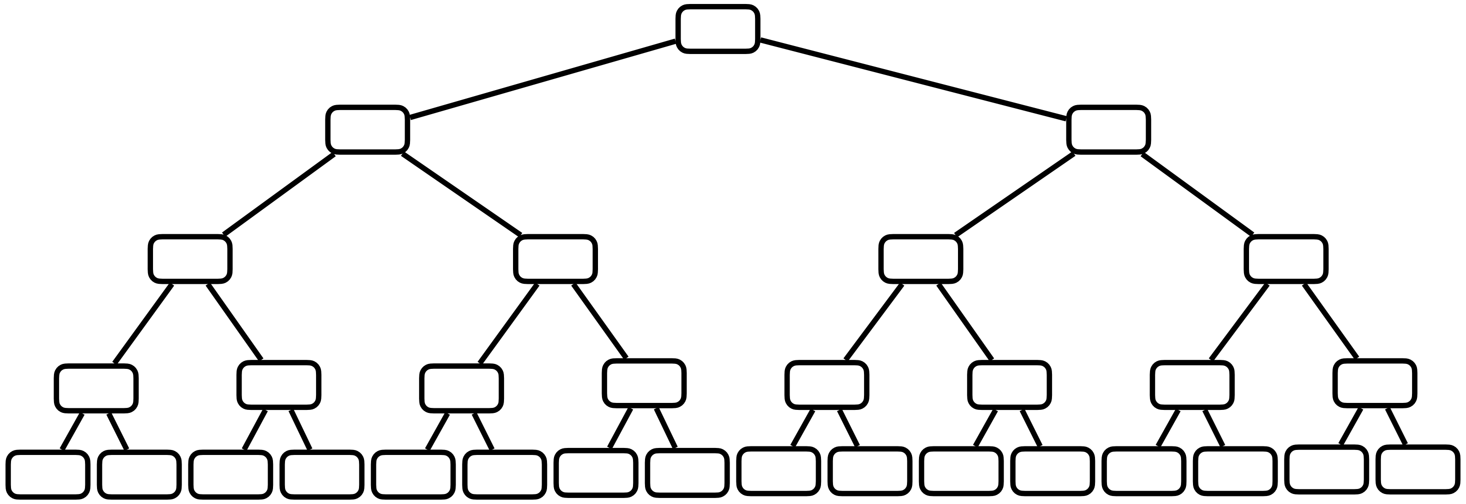
→ TCitH with GGM tree

TC-in-the-Head framework with GGM trees

TCitH with GGM trees

Step 1: Generate a replicated secret sharing [ISN89]

$x = r_1 + r_2 + \cdots + r_N$



$r_1 + r_2 + \quad \cdots \qquad \cdots \quad + r_N \quad = \quad x + \Delta_x$

TCitH with GGM trees

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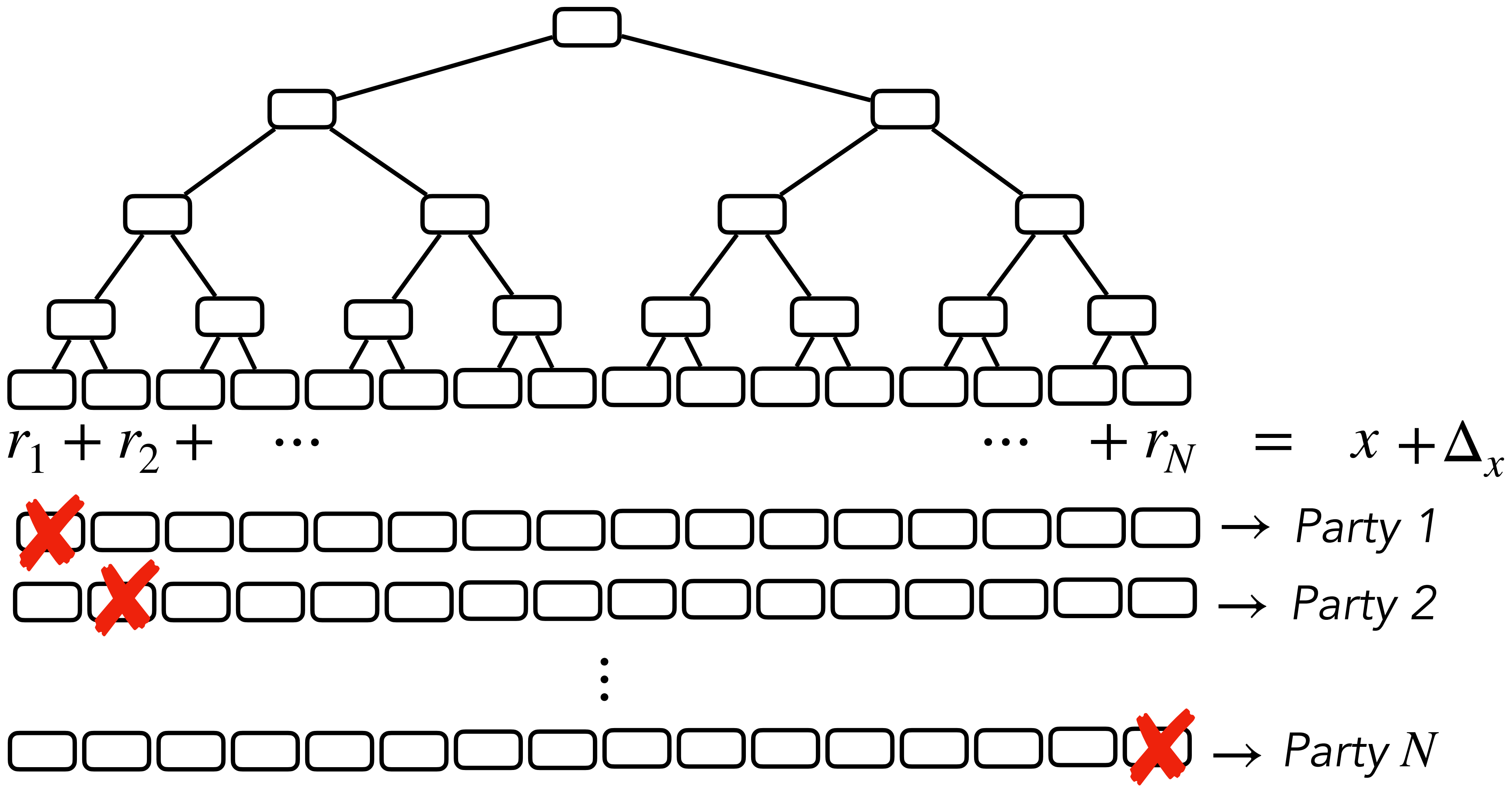

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→ *Party 1*

→ *Party 2*

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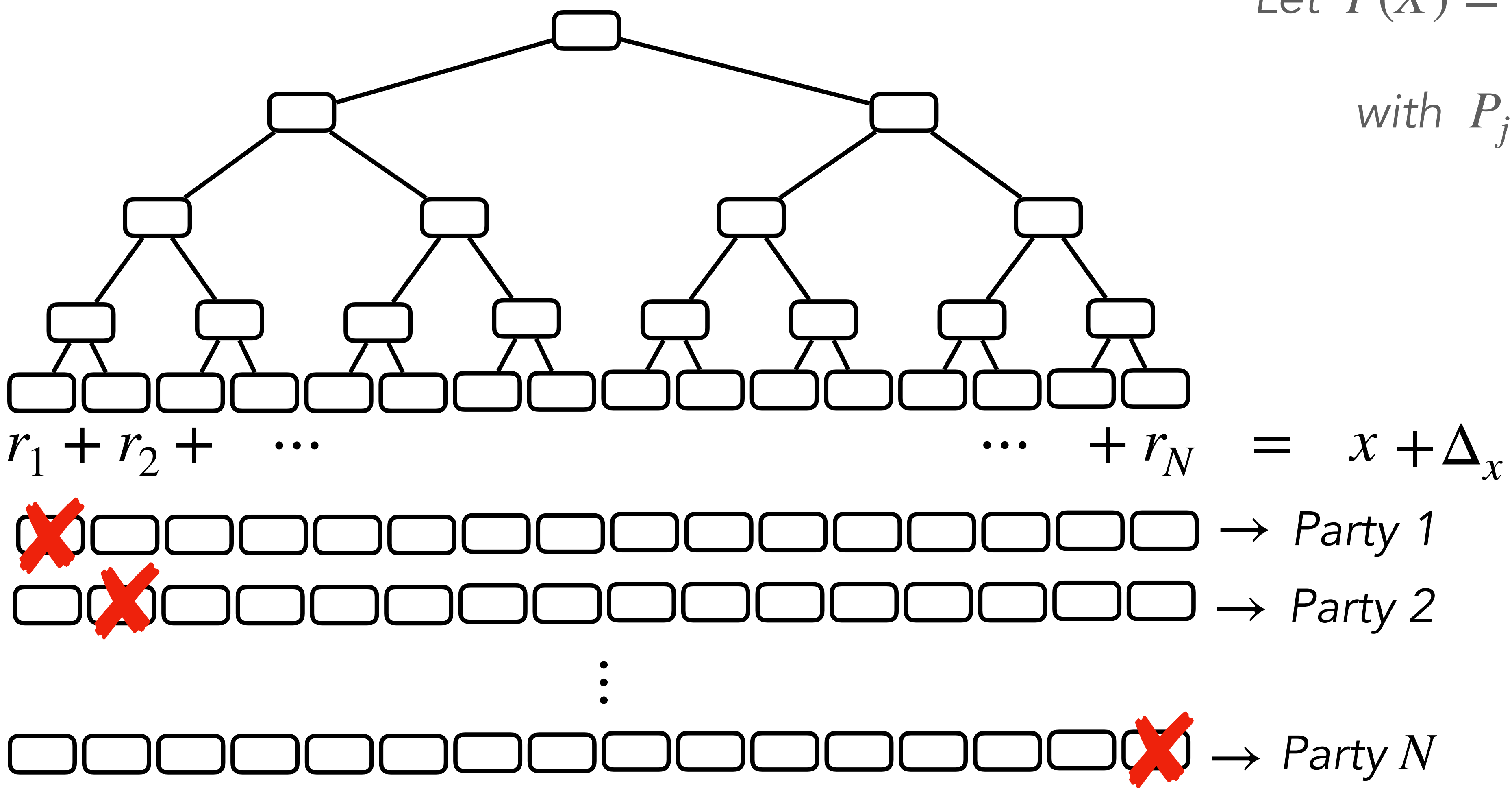

Step 2: Convert it into a Shamir's secret sharing [CDI05]

Let $P(X) = \Delta_x + \sum_j r_j P_j(X)$

with $P_j(X) = 1 - (1/e_j) \cdot X$

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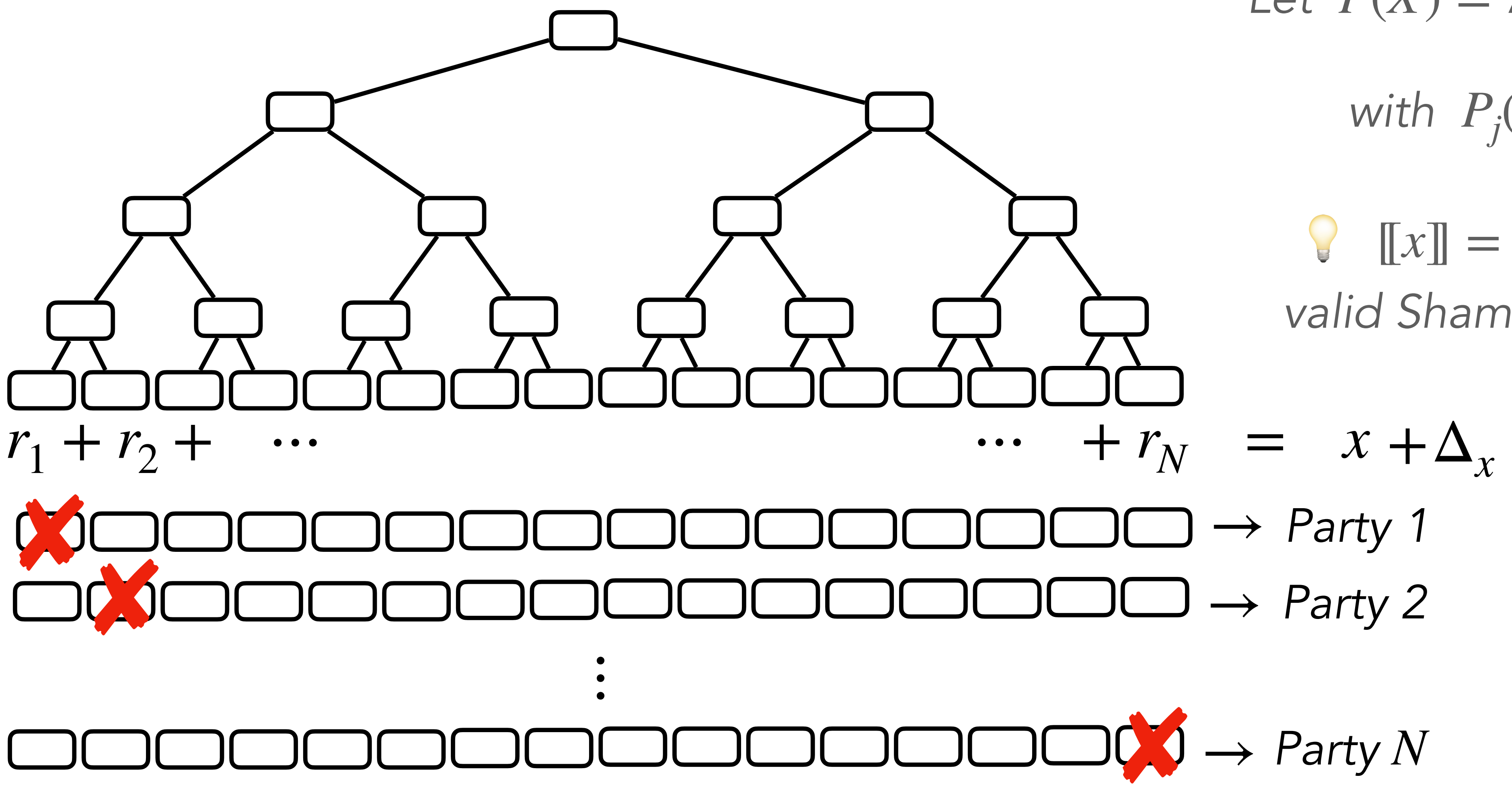

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→ *Party 2*

⋮

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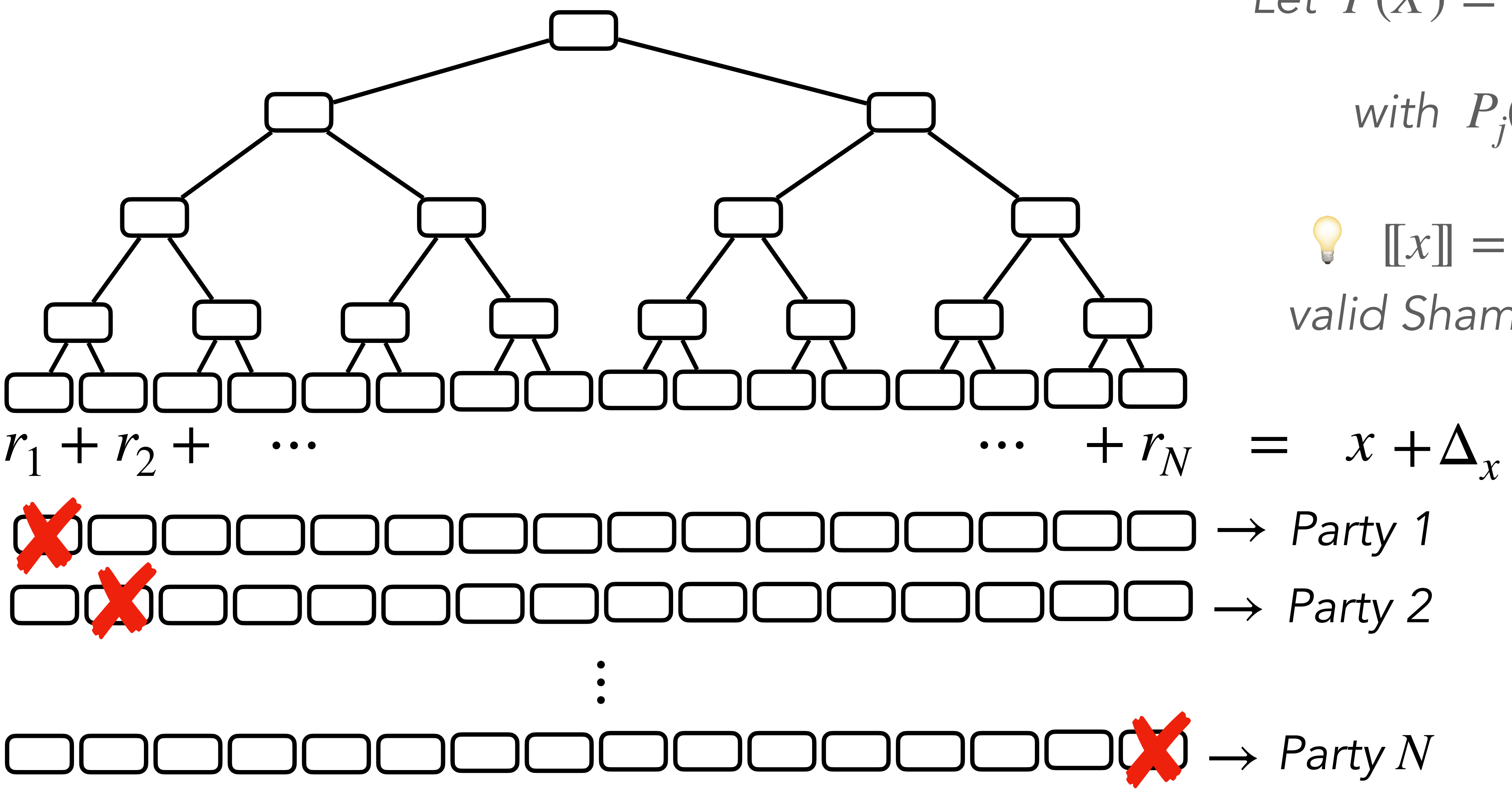
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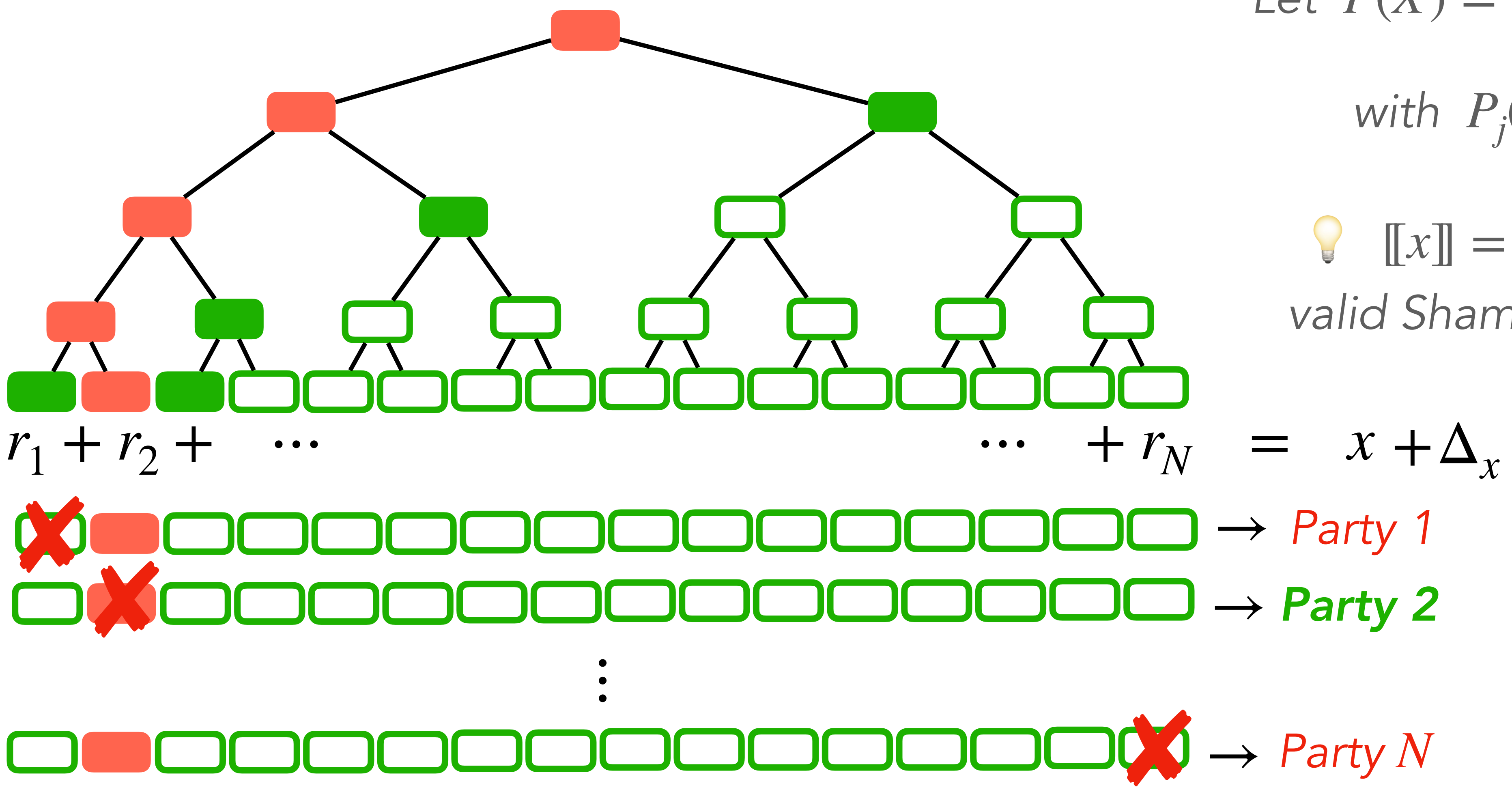

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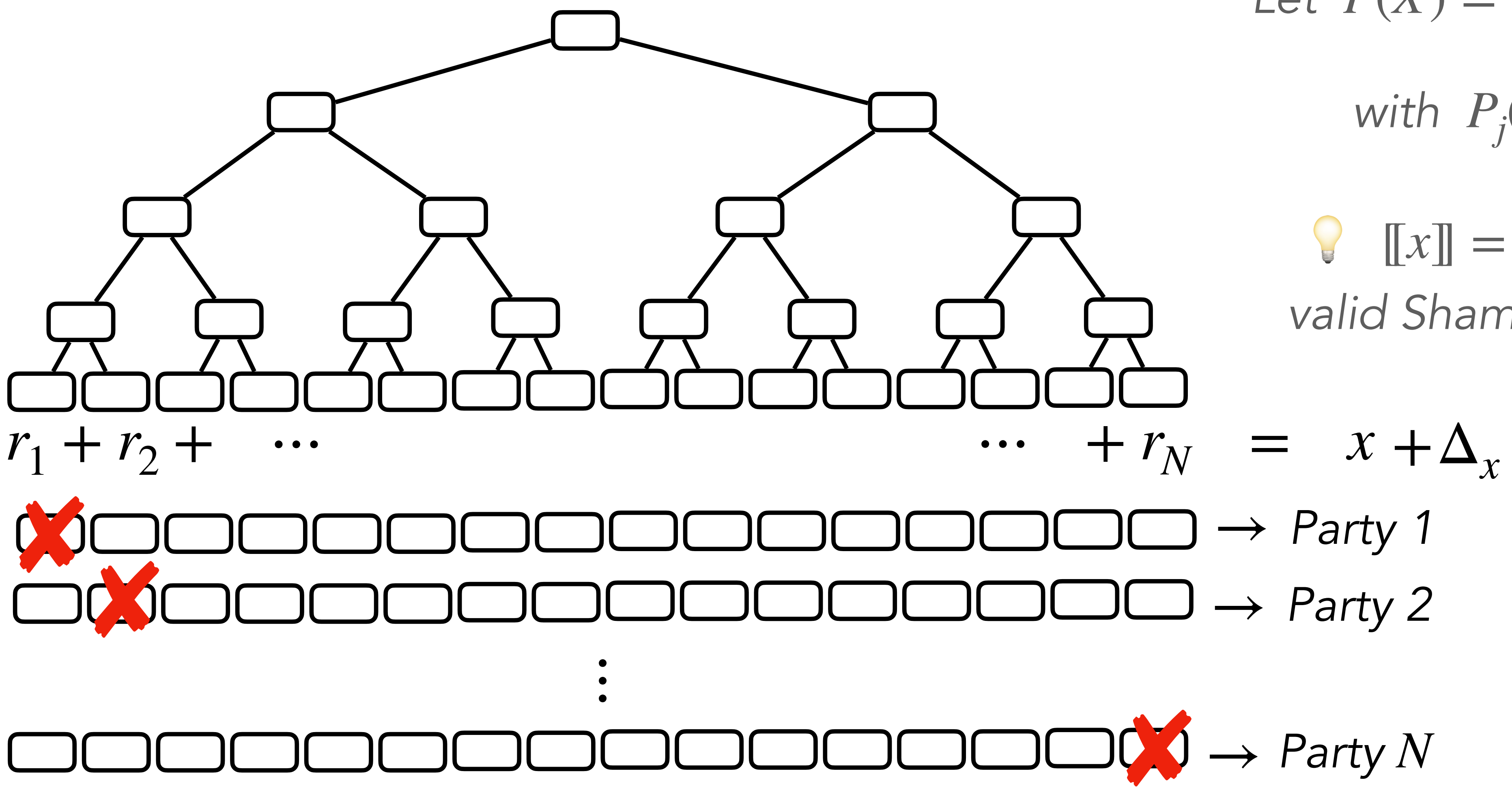

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⚒️ Can be adapted to $\ell > 1$

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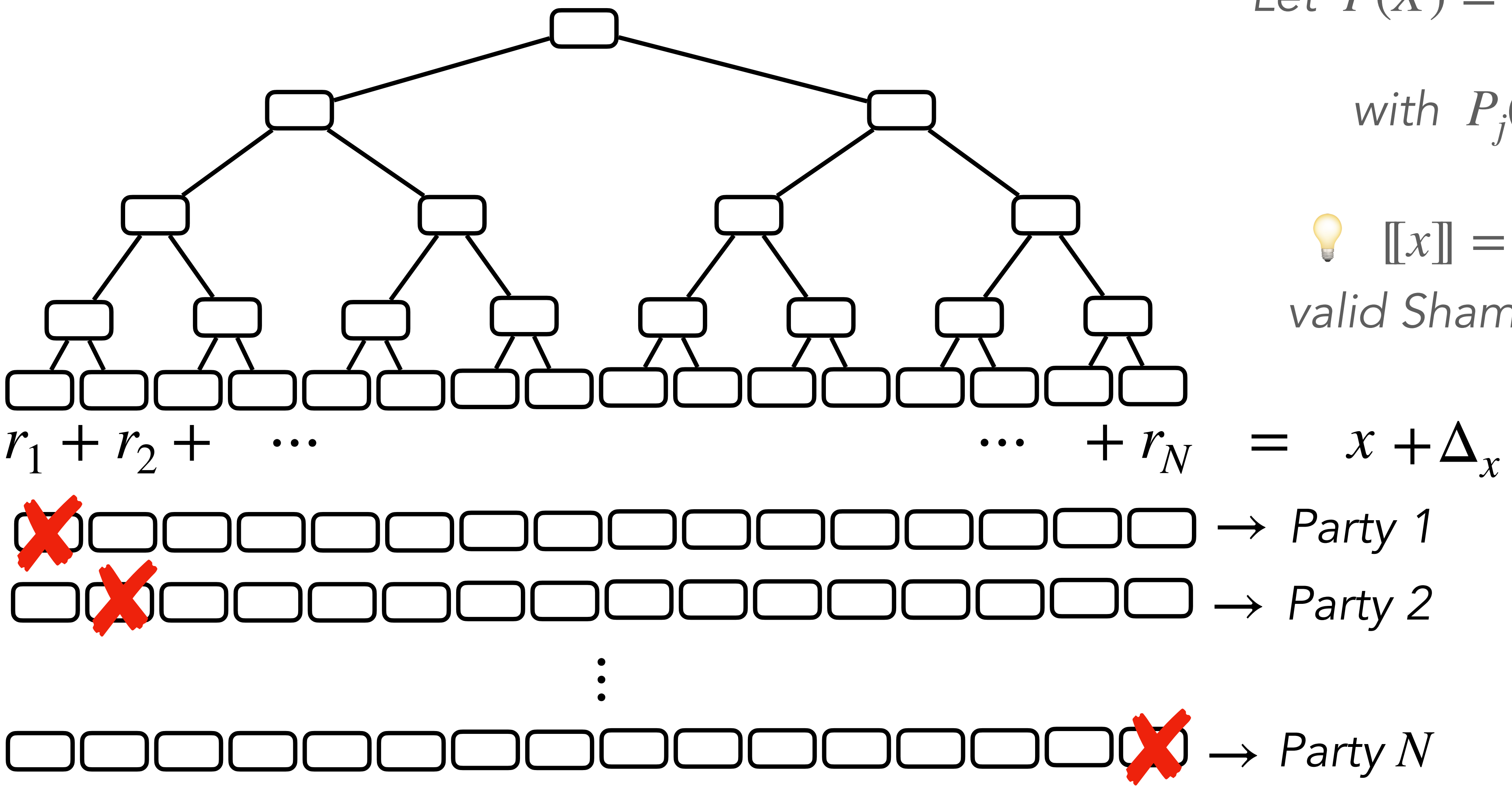

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→ *Party 2*

$\vdots$

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🌲 Size of GGM tree

TCitH with GGM trees

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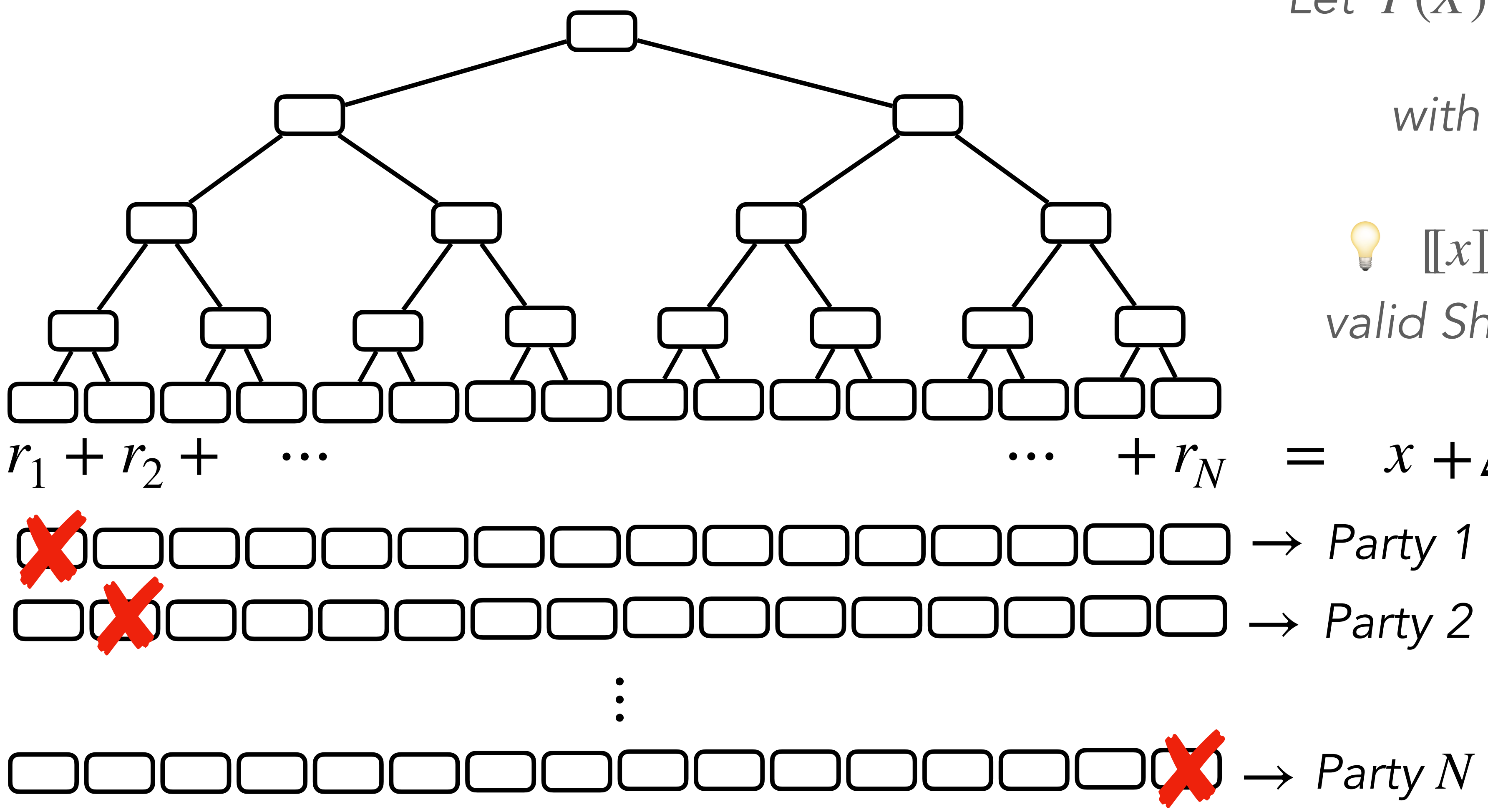
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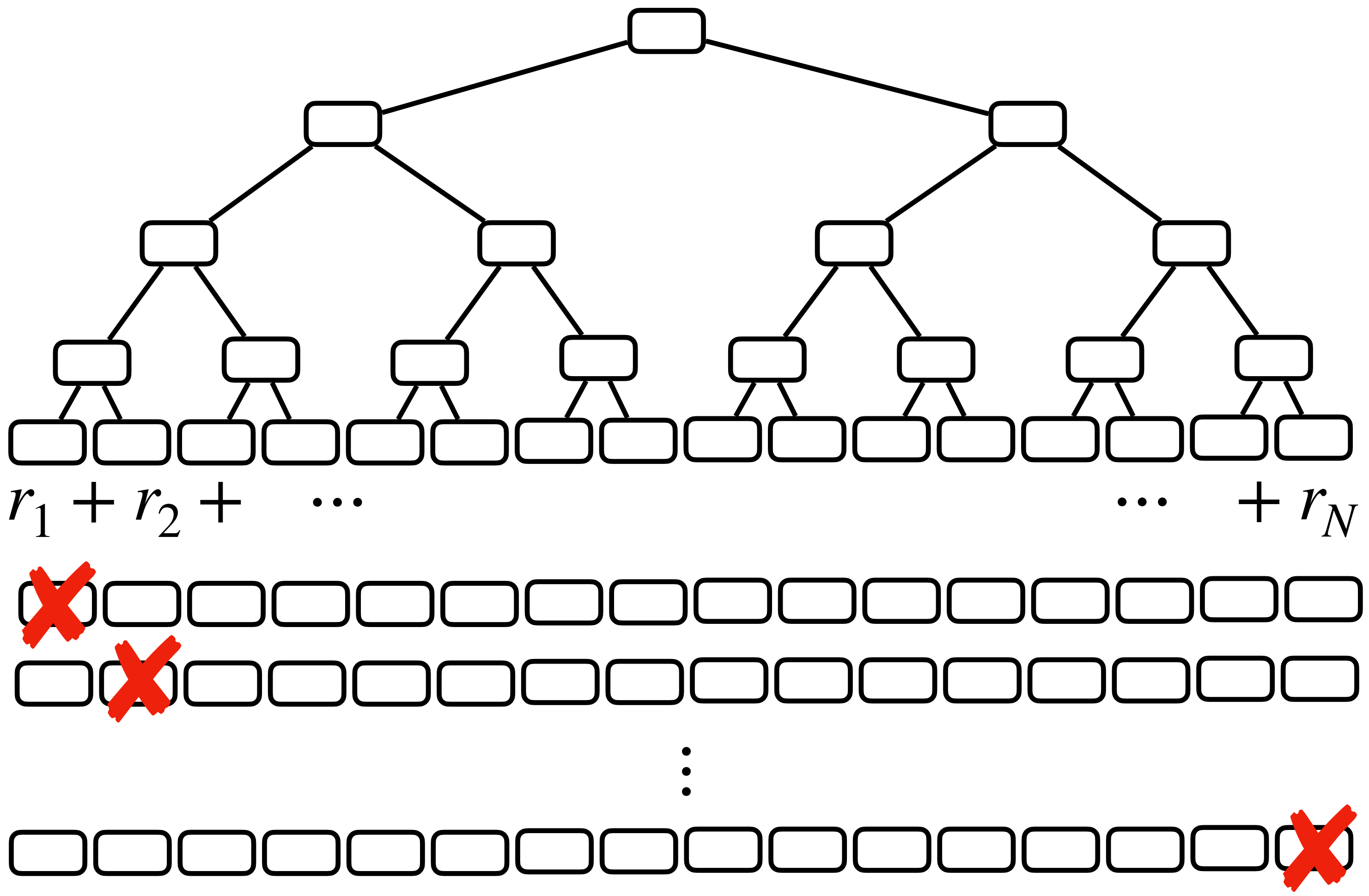

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🌲 Size of GGM tree

😇 Good soundness (only valid sharings)

🐌 Loose fast verification

Speedups for MPCitH candidates

	Additive MPCitH		TCitH (GGM tree)	
	Traditional (ms)	Hypercube (ms)	TCitH (ms)	Saving
Party emulations / repetition	N	$1 + \log_2 N$	2	

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🛠 Party emulations $= 1 + \left\lceil \frac{\log_2 N}{\log_2 |\mathbb{F}|} \right\rceil = \begin{cases} 2 & \text{if } |\mathbb{F}| \geq N \\ & \vdots \\ 1+\log_2 N & \text{if } |\mathbb{F}| = 2 \end{cases}$

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Party emulations / repetition	N	$1 + \log_2 N$	$1 + \left\lceil \frac{\log_2 N}{\log_2 \lvert\mathbb{F}\rvert} \right\rceil$	
AIMer	4.53	3.22	3.22	-0 %
Biscuit	17.71	4.65	4.24	-16 %
MIRA	384.26	20.11	9.89	-51 %
MiRitH-Ia	54.15	6.60	5.42	-18 %
MiRitH-Ib	89.50	8.66	6.66	-23 %
MQOM-31	96.41	11.27	8.74	-21 %
MQOM-251	44.11	7.56	5.97	-21 %
RYDE	12.41	4.65	4.65	-0 %
SDitH-256	78.37	7.23	5.31	-27 %
SDitH-251	19.15	7.53	6.44	-14 %

- Comparison based on a generic MPCitH library (libmpcith)
- Code for MPC protocols fetched from the submission packages

Using multiplication homomorphism & packed secret sharing

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randomness
from the verifier

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pre-committed
sharing of 0

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check $\alpha = 0$
false positive proba $1/|\mathbb{F}|$

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false positive proba $1/|\mathbb{F}|$

pre-committed
sharing of 0

randomness
from the verifier

$$\boxed{\frac{\binom{d_\alpha}{\ell}}{\binom{N}{\ell}} + p}$$

Soundness error

Using multiplication homomorphism

- Shamir's secret sharing satisfies:

$$[\![x]\!]^{(d)} \cdot [\![y]\!]^{(d)} = [\![x \cdot y]\!]^{(2d)}$$

- Simple protocol to verify polynomial constraints
  - w valid $\Leftrightarrow f_1(w) = 0, \; \dots, \; f_m(w) = 0$
  - parties locally compute

$$\underbrace{[\![\alpha]\!]}_{\substack{\text{check } \alpha = 0 \\ \text{false positive proba } 1/|\mathbb{F}|}} = \underbrace{[\![v]\!]}_{\substack{\text{pre-committed} \\ \text{sharing of 0}}} + \sum_{j=1}^{m} \underbrace{\gamma_j}_{\substack{\text{randomness} \\ \text{from the verifier}}} \cdot f_j([\![w]\!])$$

Here:

$$\boxed{\frac{\dbinom{\overset{\ell \cdot \deg f_j}{d_\alpha}}{\ell}}{\dbinom{N}{\ell}} + \overset{\left(\frac{1}{|\mathbb{F}|}\right)^{\#\alpha}}{p}}$$

Soundness error

Shorter signatures for MPCitH-based candidates

	Original Size	***Our Variant***	***Saving***
Biscuit	4 758 B	4 048 B	-15 %
MIRA	5 640 B	5 340 B	-5 %
MiRitH-Ia	5 665 B	4 694 B	-17 %
MiRitH-Ib	6 298 B	5 245 B	-17 %
MQOM-31	6 328 B	4 027 B	-37 %
MQOM-251	6 575 B	4 257 B	-35 %
RYDE	5 956 B	5 281 B	-11 %
SDitH	8 241 B	7 335 B	-27 %
MQ over GF(4)	8 609 B	3 858 B	-55 %
SD over GF(2)	11 160 B	7 354 B	-34 %
SD over GF(2)	12 066 B	6 974 B	-42 %

* $N = 256$

Shorter signatures for MPCitH-based candidates

	Original Size	*Our Variant*	*Saving*
Biscuit	4 758 B	3 431 B	
MIRA	5 640 B	4 314 B	
MiRitH-Ia	5 665 B	3 873 B	
MiRitH-Ib	6 298 B	4 250 B	
MQOM-31	6 328 B	3 567 B	
MQOM-251	6 575 B	3 418 B	
RYDE	5 956 B	4 274 B	
SDitH	8 241 B	5 673 B	

MQ over GF(4)	8 609 B	3 301 B	
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* $N = 256$ * $N = 2048$

Shorter signatures for MPCitH-based candidates

Two very recent works :

- Baum, Beullens, Mukherjee, Orsini, Ramacher, Rechberger, Roy, Scholl. *One Tree to Rule Them All: Optimizing GGM Trees and OWFs for Post-Quantum Signatures.* https://ia.cr/2024/490
  - General techniques to reduce the size of GGM trees
  - **Apply to TCitH-GGM** (gain of ~500 B at 128-bit security)

- Bidoux, Feneuil, Gaborit, Neveu, Rivain. *Dual Support Decomposition in the Head: Shorter Signatures from Rank SD and MinRank.* https://ia.cr/2024/541
  - New MPC protocols for TCitH / VOLEitH signatures based on **MinRank & Rank SD**

Using packed secret sharing

- Shamir's secret sharing can be packed
  - $P(\omega_1) = x_1 , \quad \ldots , \; P(\omega_s) = x_s$
  - $P(\omega_{s+1}) = r_1 , \; \ldots , P(\omega_{s+\ell}) = r_\ell$
  - $[\![x]\!]_1 = P(e_1) , \; \ldots , [\![x]\!]_N = P(e_N)$

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- $[\![x]\!] \cdot [\![y]\!] =$ sharing of $(x_1, \ldots, x_s) \circ (y_1, \ldots, y_s)$

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Here: $(\ell + s - 1) \cdot \deg f_j$

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  ⇒ interesting for statements with "medium size" witness

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- Packed sharing & Merkle trees ≈ ÷ witness size by s

  ⇒ interesting for statements with "medium size" witness
- E.g. an ISIS statement $\vec{t} = A \cdot \vec{e}$ with $\| \vec{e} \|_\infty \leq \beta$

Here: $(\ell + s - 1) \cdot \deg f_j$

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Soundness error

TCitH-GGM vs. TCitH-MT

TCitH-GGM	*TCitH-MT*
🎄 Smaller tree	🌲 Larger tree (~x2)

TCitH-GGM vs. TCitH-MT

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❌📦 No advantage of packed sharing	📦 Takes advantage of packed sharing

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☑ Naturally enforce degree of committed sharings	🔁 Need degree enforcing commitment (+1 round)

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TCitH-GGM	***TCitH-MT***
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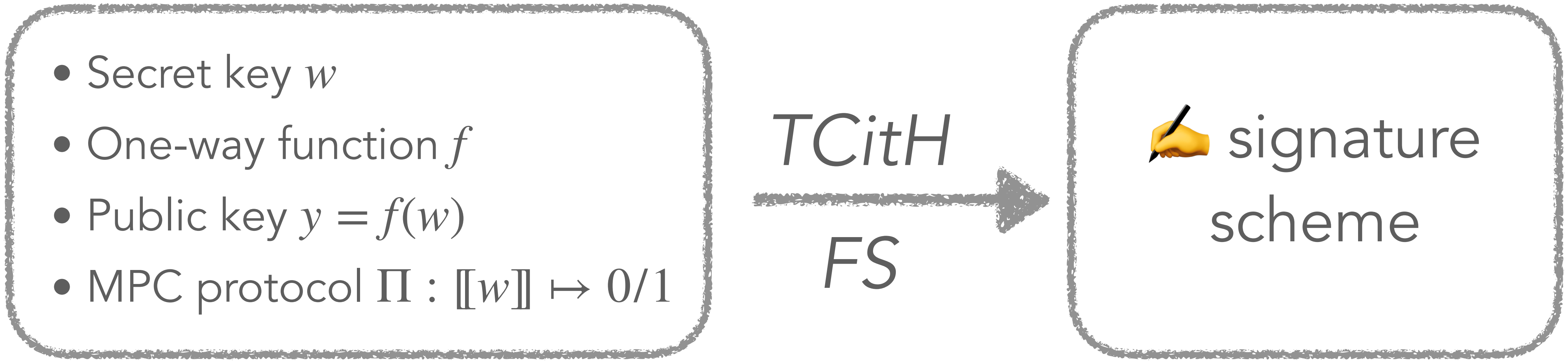
☑️ Naturally enforce degree of committed sharings	🔁 Need degree enforcing commitment (+1 round)
🎯 Better for "small-size" statements	🎯 Better for "medium-size" statements

Application: post-quantum ring signatures

Post-quantum ring signatures

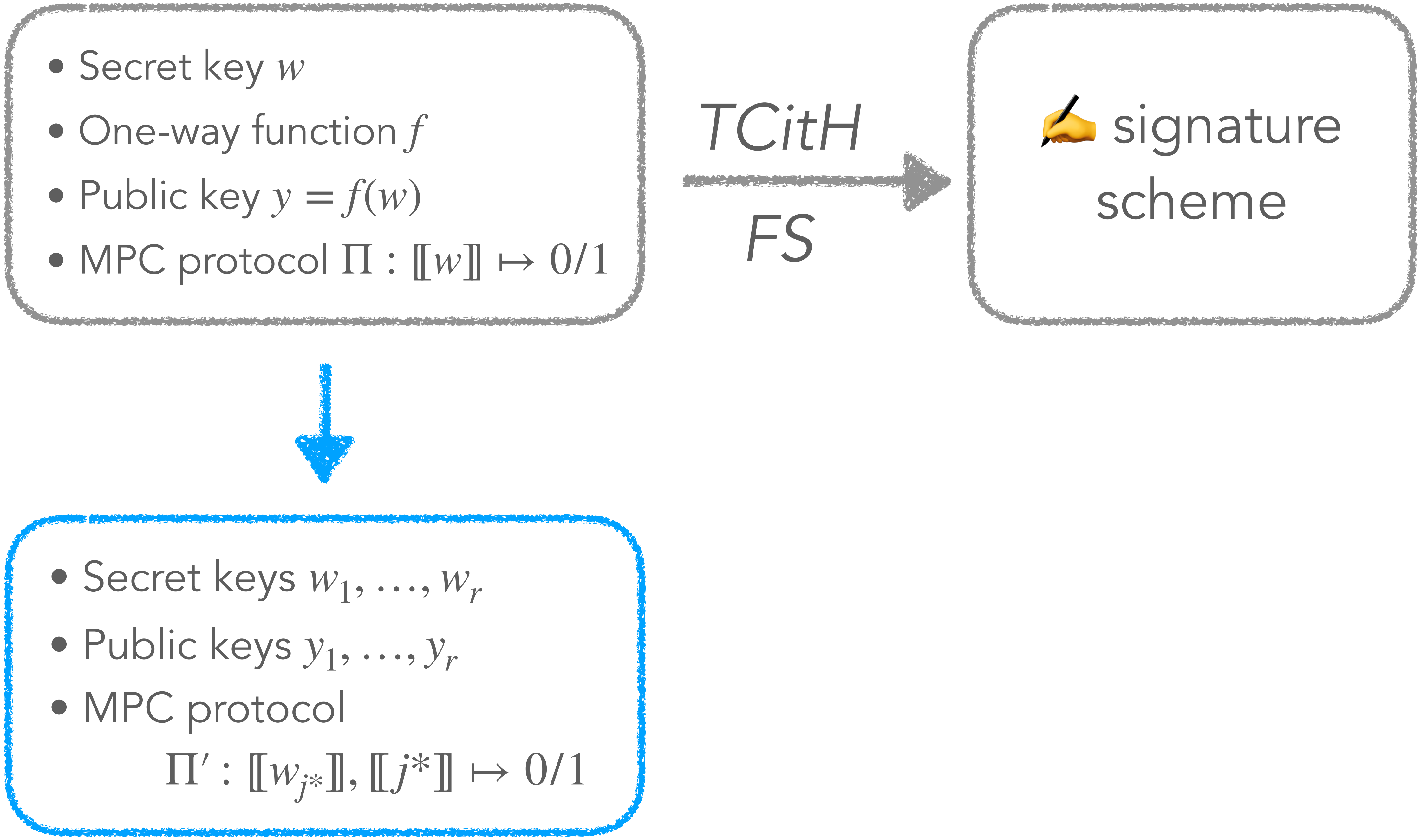


- Secret key w
- One-way function f
- Public key $y = f(w)$
- MPC protocol $\Pi : [\![w]\!] \mapsto 0/1$

$\xrightarrow[FS]{TCitH}$

✍️ signature scheme

Post-quantum ring signatures



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TCitH

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✍ signature scheme

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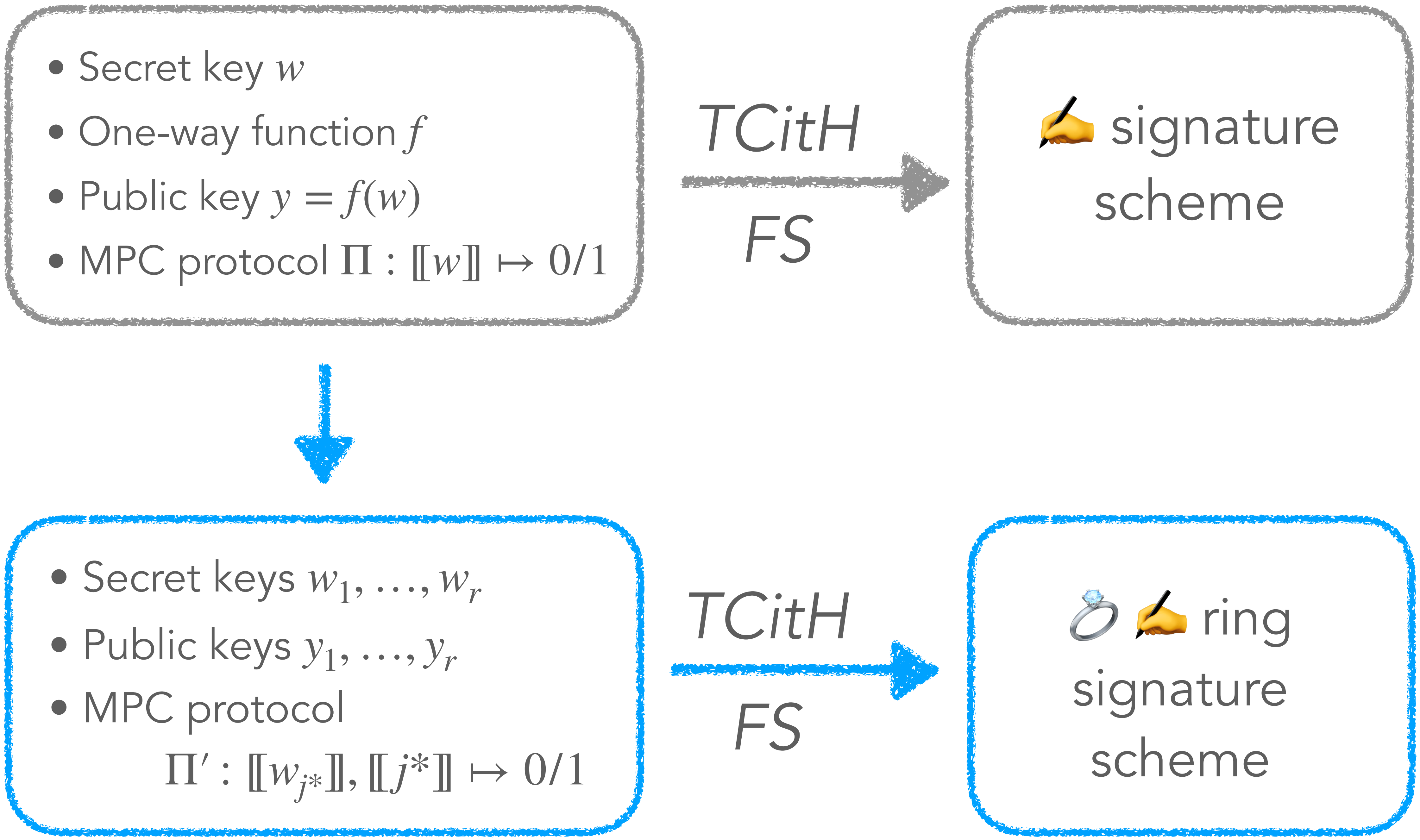
Post-quantum ring signatures



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💍 ✍ ring signature scheme

Post-quantum ring signatures

💡 Idea:

- One-hot encoding of $j*$

$$s = (0,\ldots,0,\, s_{j*} := 1,\, 0,\ldots,0)$$

Post-quantum ring signatures

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Post-quantum ring signatures

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🤔 Problem: including $\llbracket s \rrbracket$ to the witness ⇒ $\mathcal{O}(r)$ signature size

Post-quantum ring signatures

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🤔 Problem: including $\llbracket s \rrbracket$ to the witness $\Rightarrow \mathcal{O}(r)$ signature size

🛠️ Solution: $\llbracket s^{(1)} \rrbracket, \ldots, \llbracket s^{(d)} \rrbracket$ s.t. $s = s^{(1)} \otimes \cdots \otimes s^{(d)}$

$\Rightarrow \mathcal{O}\left(d \sqrt[d]{r}\,\right)$ signature size $\Rightarrow \mathcal{O}(\log r)$

Post-quantum ring signatures

Protocol Π'

Input: $[\![w]\!], [\![s^{(1)}]\!], \ldots, [\![s^{(d)}]\!]$

Post-quantum ring signatures

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🛠️ Simple
MPC protocol

Post-quantum ring signatures

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🛠 Simple MPC protocol

⚠ Π must be adapted to use $[\![y_{j^*}]\!]$ instead of y_{j^*}

Post-quantum ring signatures

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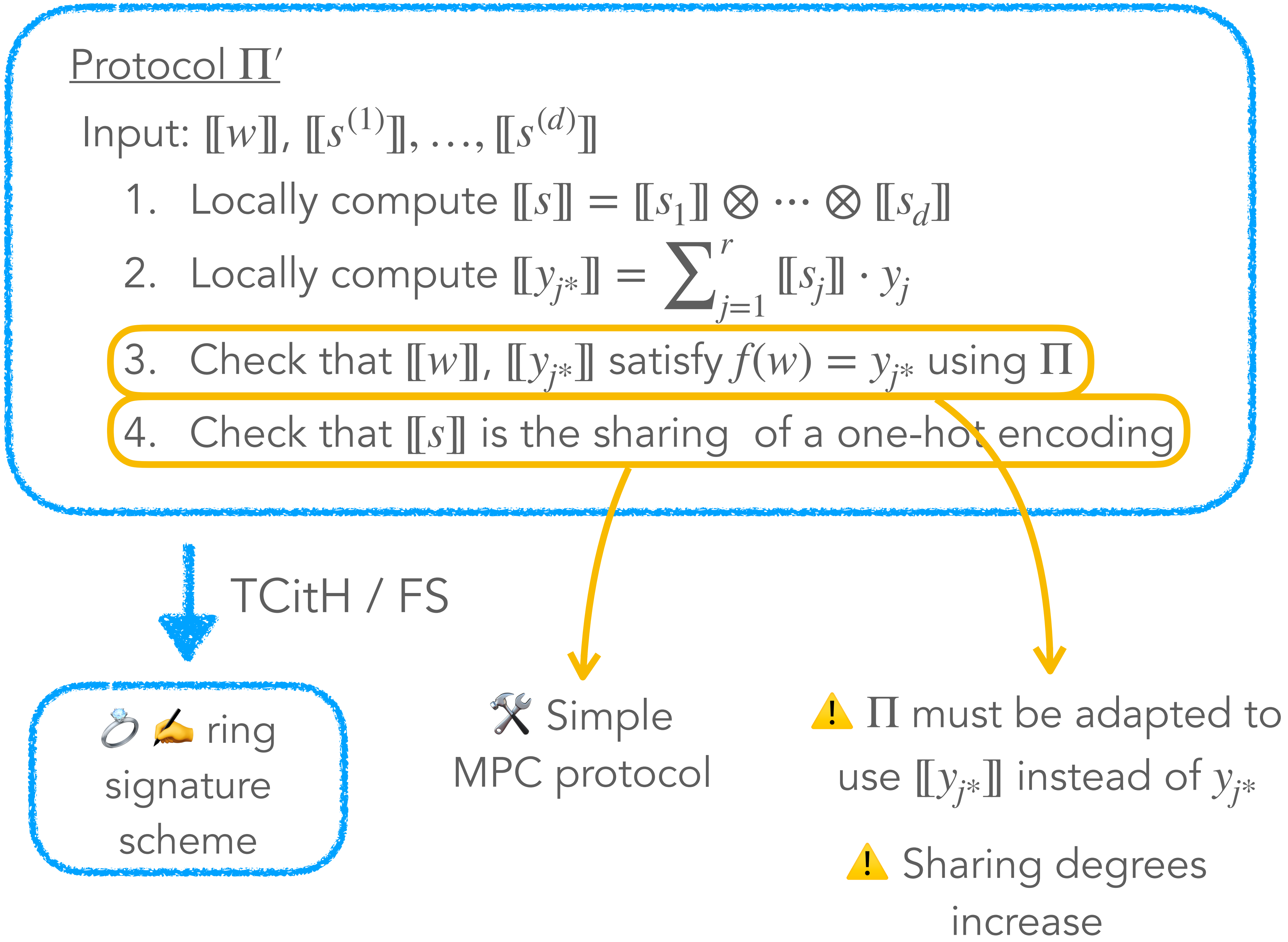
Post-quantum ring signatures



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TCitH / FS

💍 ✍️ ring signature scheme

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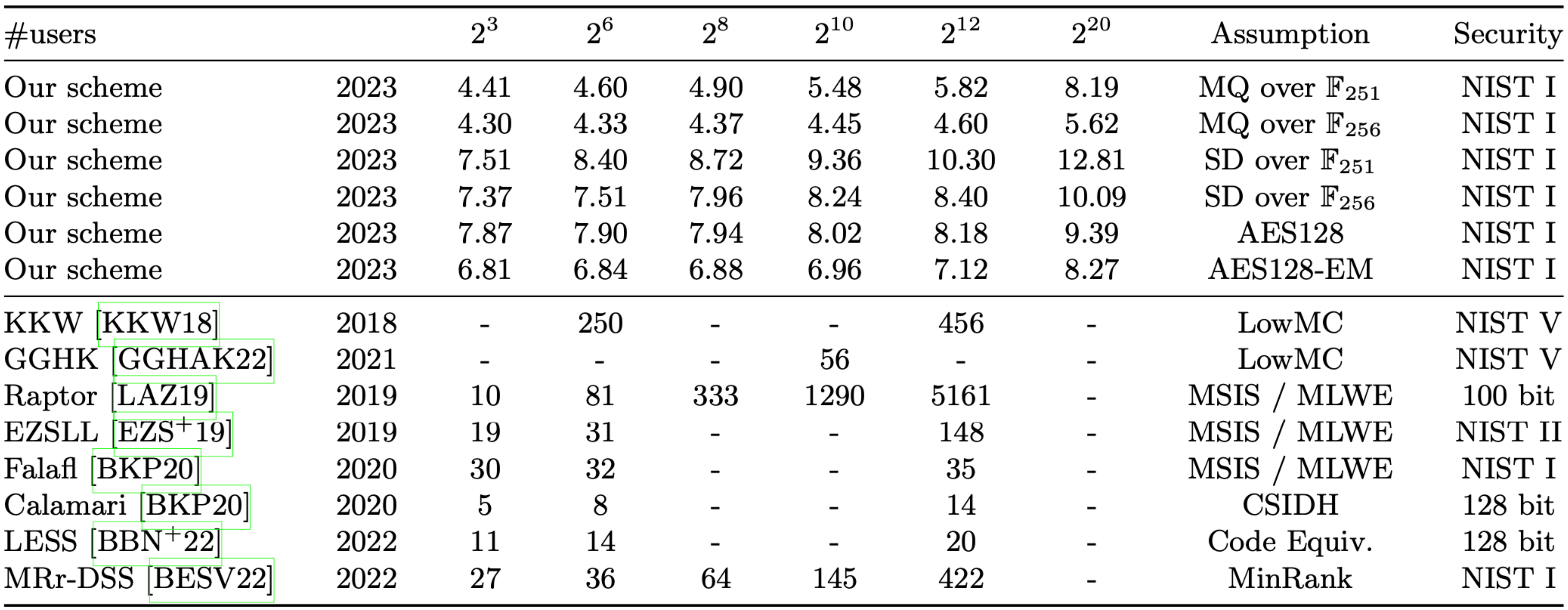

#users		2^3	2^6	2^8	2^{10}	2^{12}	2^{20}	Assumption	Security
Our scheme	2023	4.41	4.60	4.90	5.48	5.82	8.19	MQ over $\mathbb{F}_{251}$	NIST I
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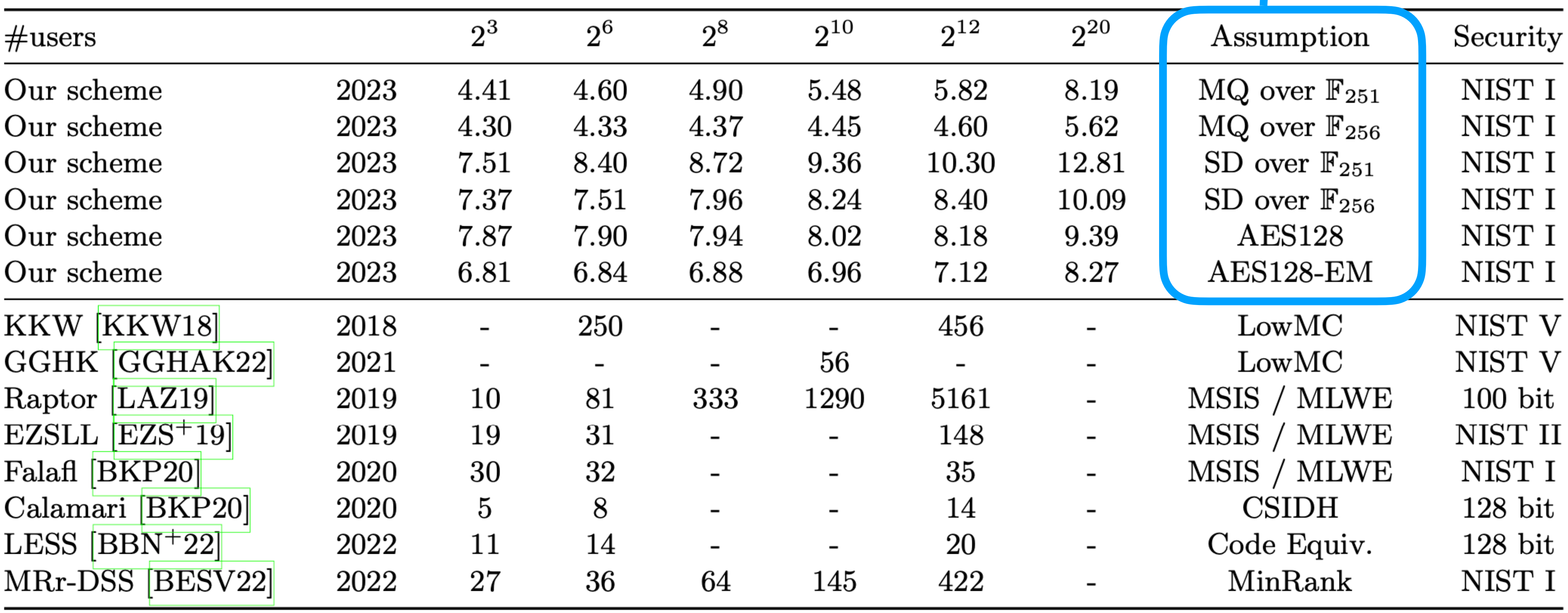
Raptor [LAZ19]	2019	10	81	333	1290	5161	-	MSIS / MLWE	100 bit
EZSLL [EZS+19]	2019	19	31	-	-	148	-	MSIS / MLWE	NIST II
Falafl [BKP20]	2020	30	32	-	-	35	-	MSIS / MLWE	NIST I
Calamari [BKP20]	2020	5	8	-	-	14	-	CSIDH	128 bit
LESS [BBN+22]	2022	11	14	-	-	20	-	Code Equiv.	128 bit
MRr-DSS [BESV22]	2022	27	36	64	145	422	-	MinRank	NIST I

Post-quantum ring signatures

Application to MQ, SD, AES



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Post-quantum ring signatures

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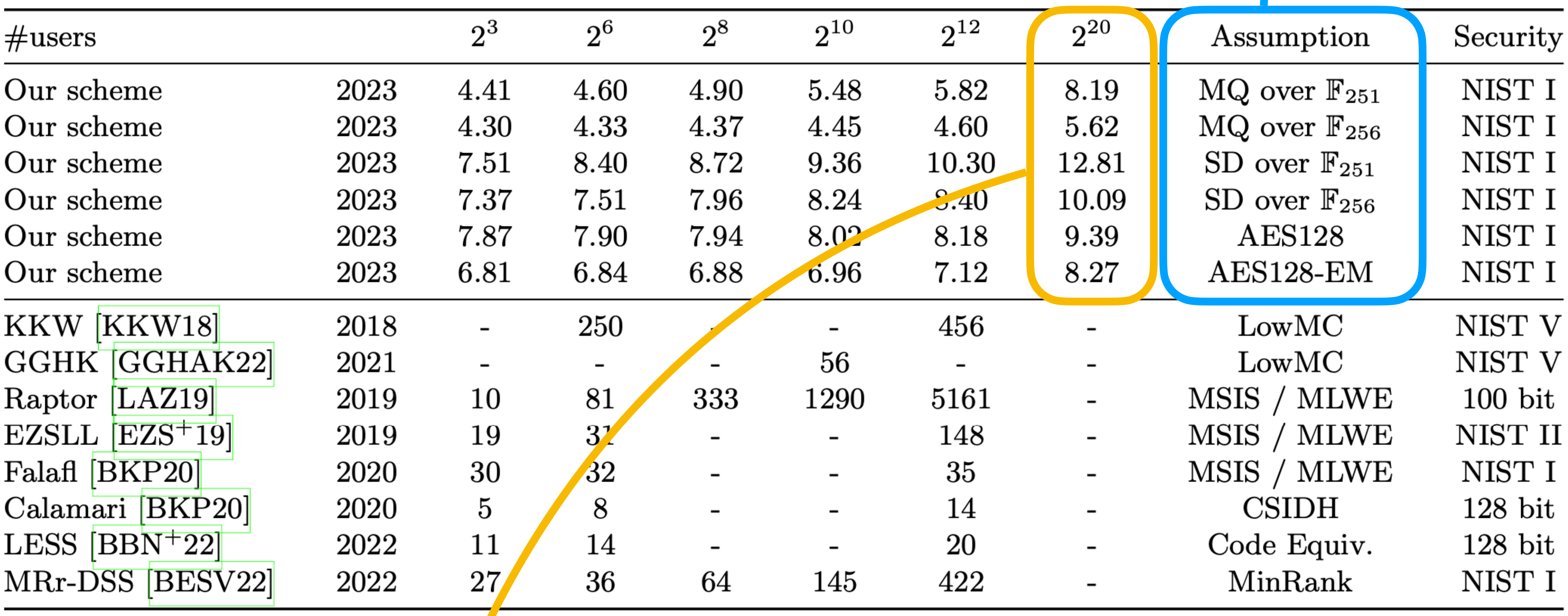

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Size range: 5–13 kB for |ring|=2^{20}

Post-quantum ring signatures

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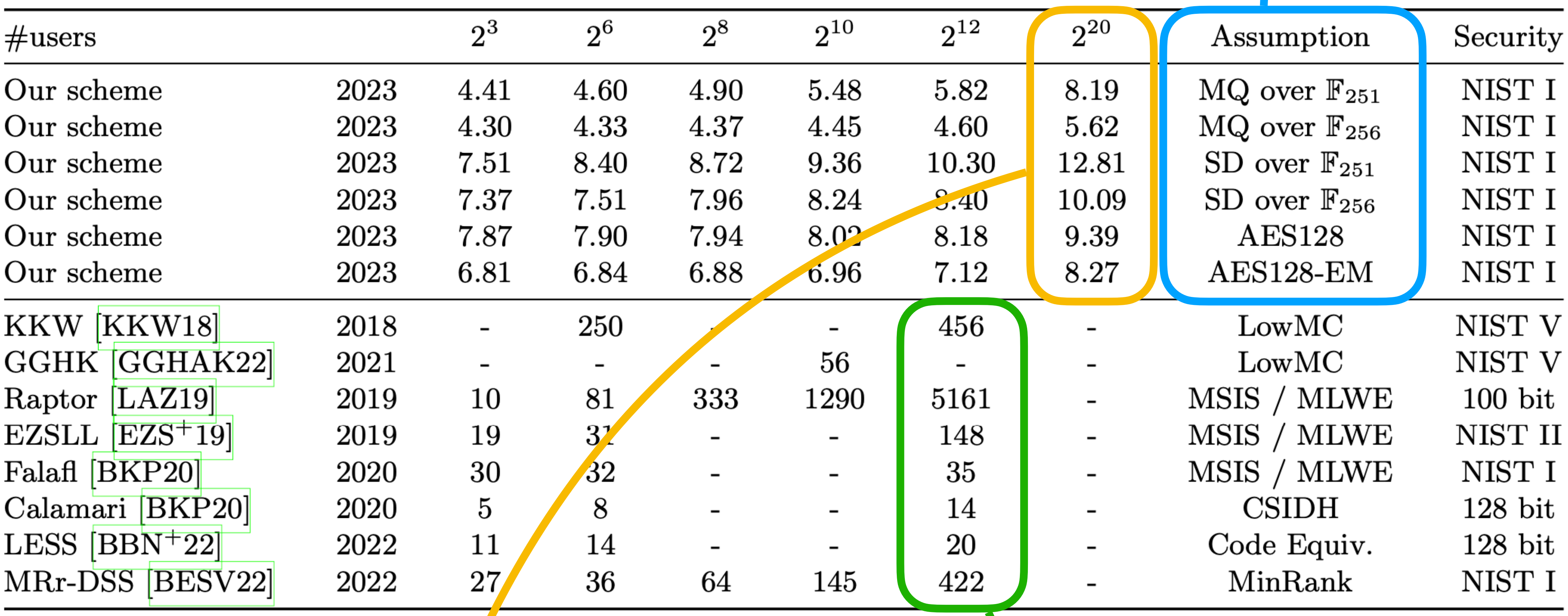

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GGHK [GGHAK22]	2021	-	-	-	56	-	-	LowMC	NIST V
Raptor [LAZ19]	2019	10	81	333	1290	5161	-	MSIS / MLWE	100 bit
EZSLL [EZS+19]	2019	19	31	-	-	148	-	MSIS / MLWE	NIST II
Falafl [BKP20]	2020	30	32	-	-	35	-	MSIS / MLWE	NIST I
Calamari [BKP20]	2020	5	8	-	-	14	-	CSIDH	128 bit
LESS [BBN+22]	2022	11	14	-	-	20	-	Code Equiv.	128 bit
MRr-DSS [BESV22]	2022	27	36	64	145	422	-	MinRank	NIST I

Size range: 5–13 kB

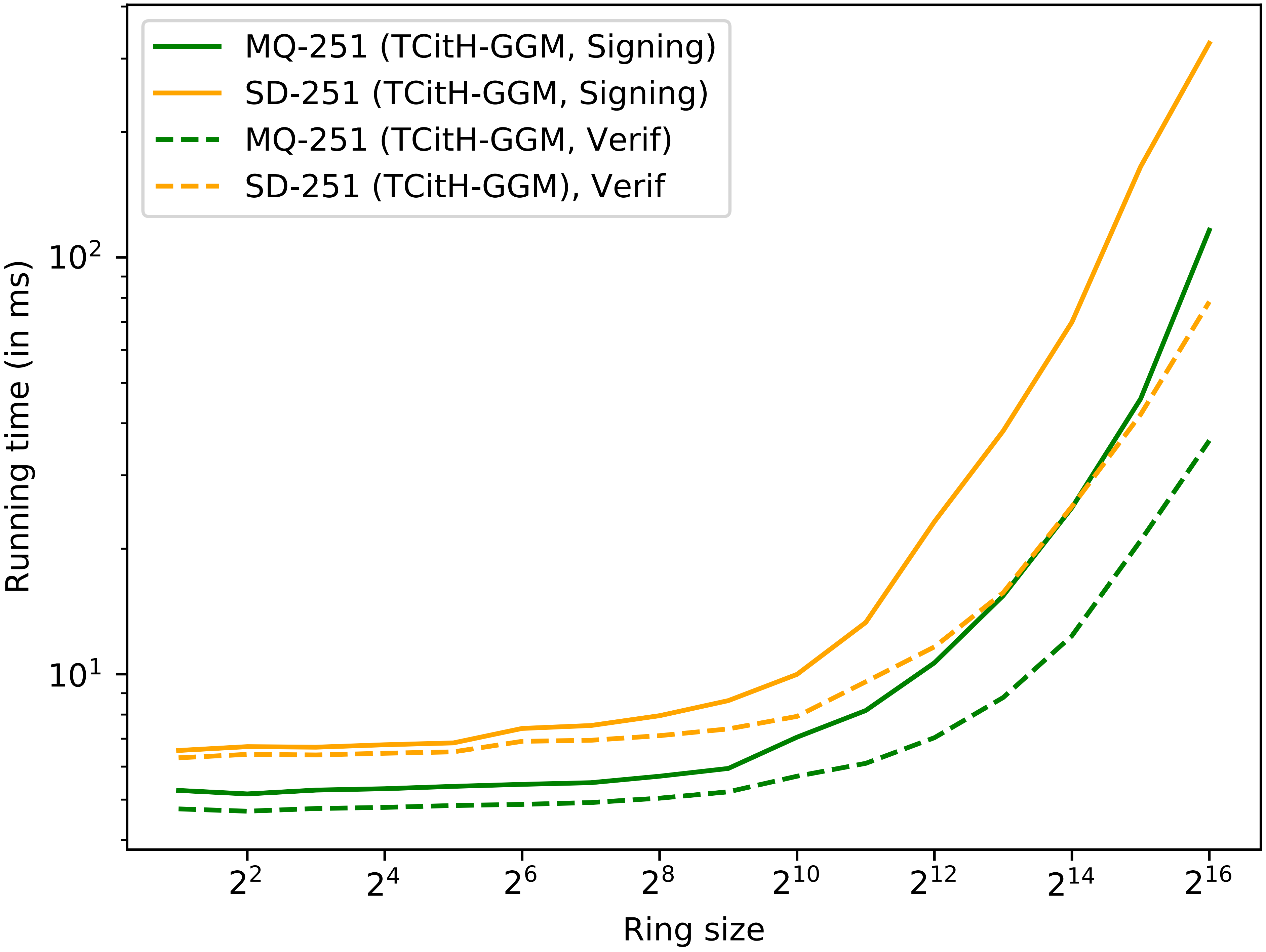
for |ring|=2^{20}

Previous works:

$\geq$ 14 kB for |ring|=2^{10}

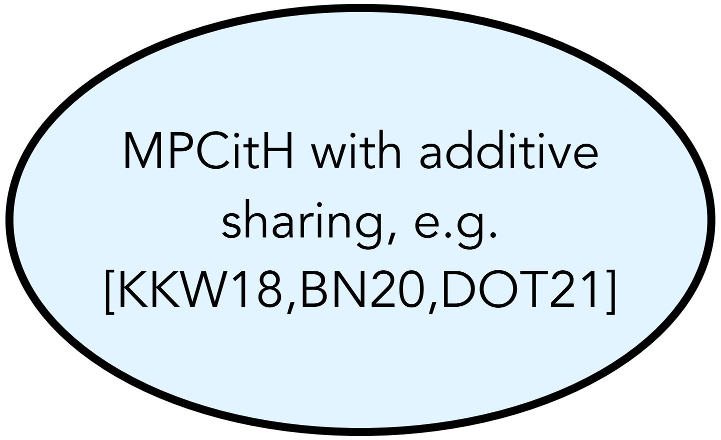
no / slow implementations

Post-quantum ring signatures



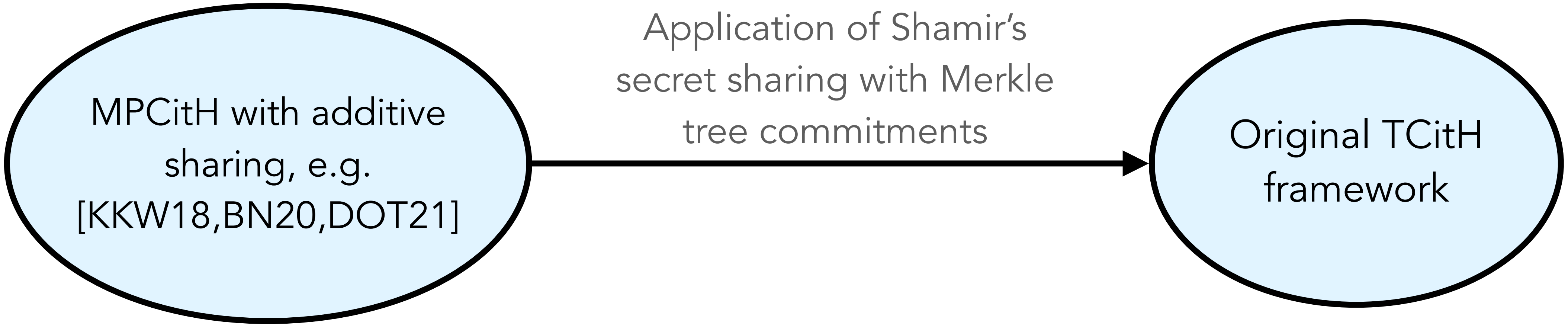
Relation to other proof systems

Connections to other proof systems

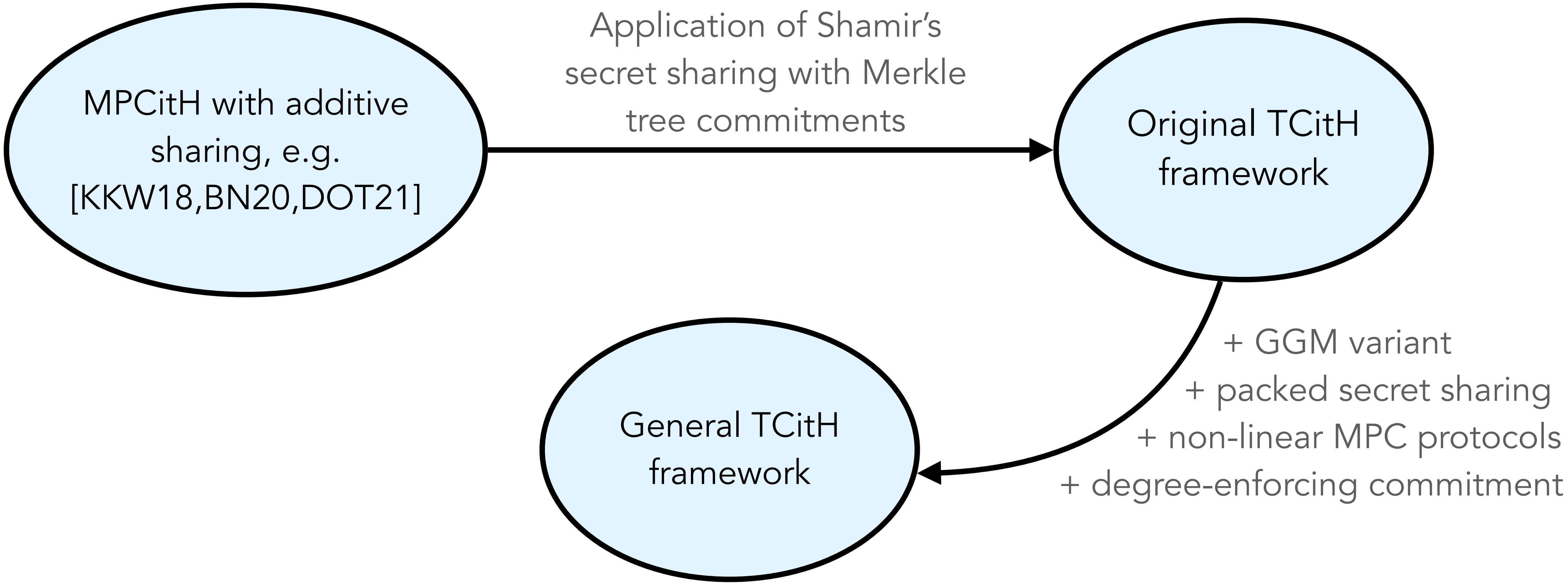


MPCitH with additive sharing, e.g. [KKW18,BN20,DOT21]

Connections to other proof systems


MPCitH with additive sharing, e.g. [KKW18,BN20,DOT21]
Application of Shamir's secret sharing with Merkle tree commitments
Original TCitH framework

Connections to other proof systems



MPCitH with additive sharing, e.g. [KKW18,BN20,DOT21]

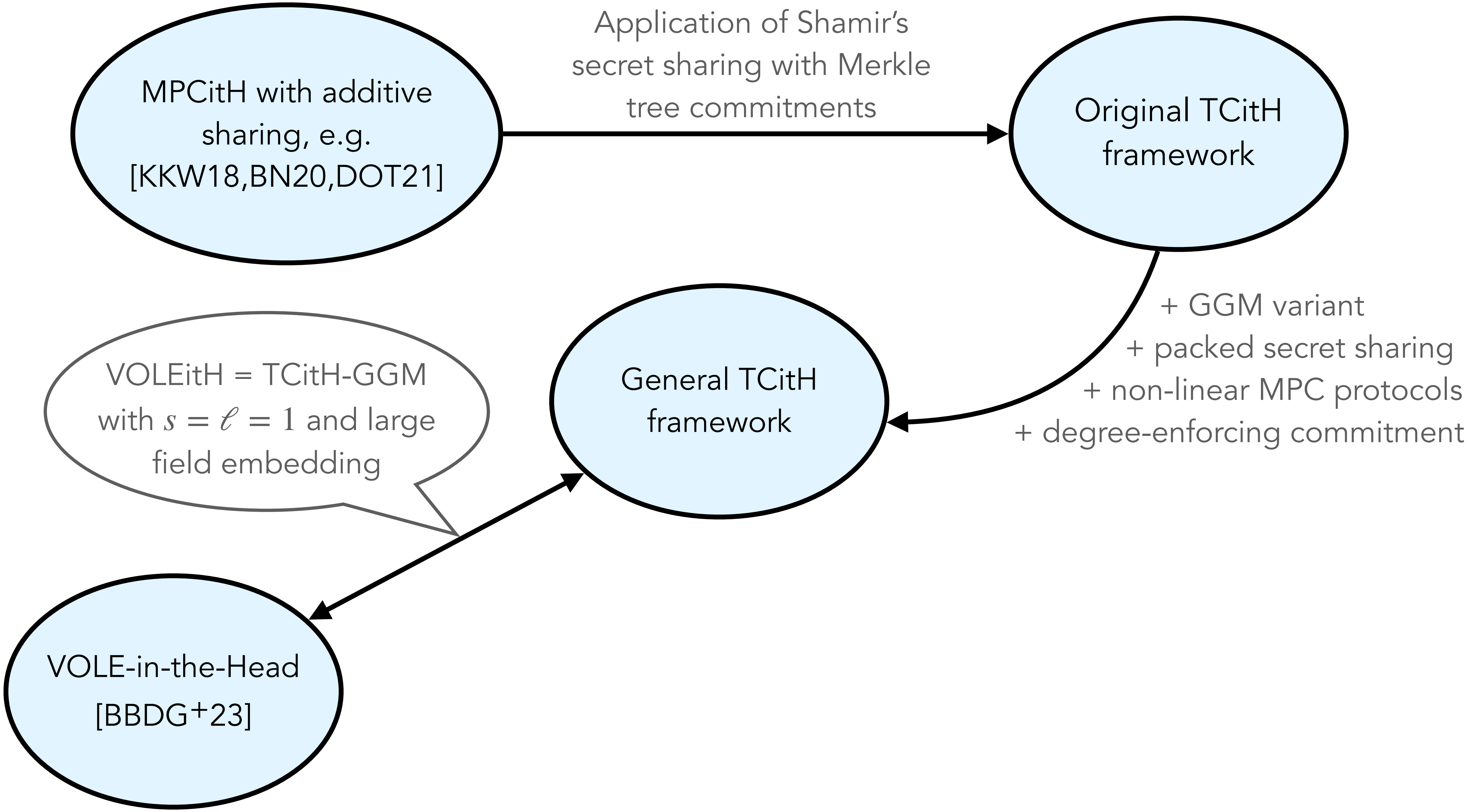
Application of Shamir's secret sharing with Merkle tree commitments

Original TCitH framework

+ GGM variant
+ packed secret sharing
+ non-linear MPC protocols
+ degree-enforcing commitment

General TCitH framework

Connections to other proof systems



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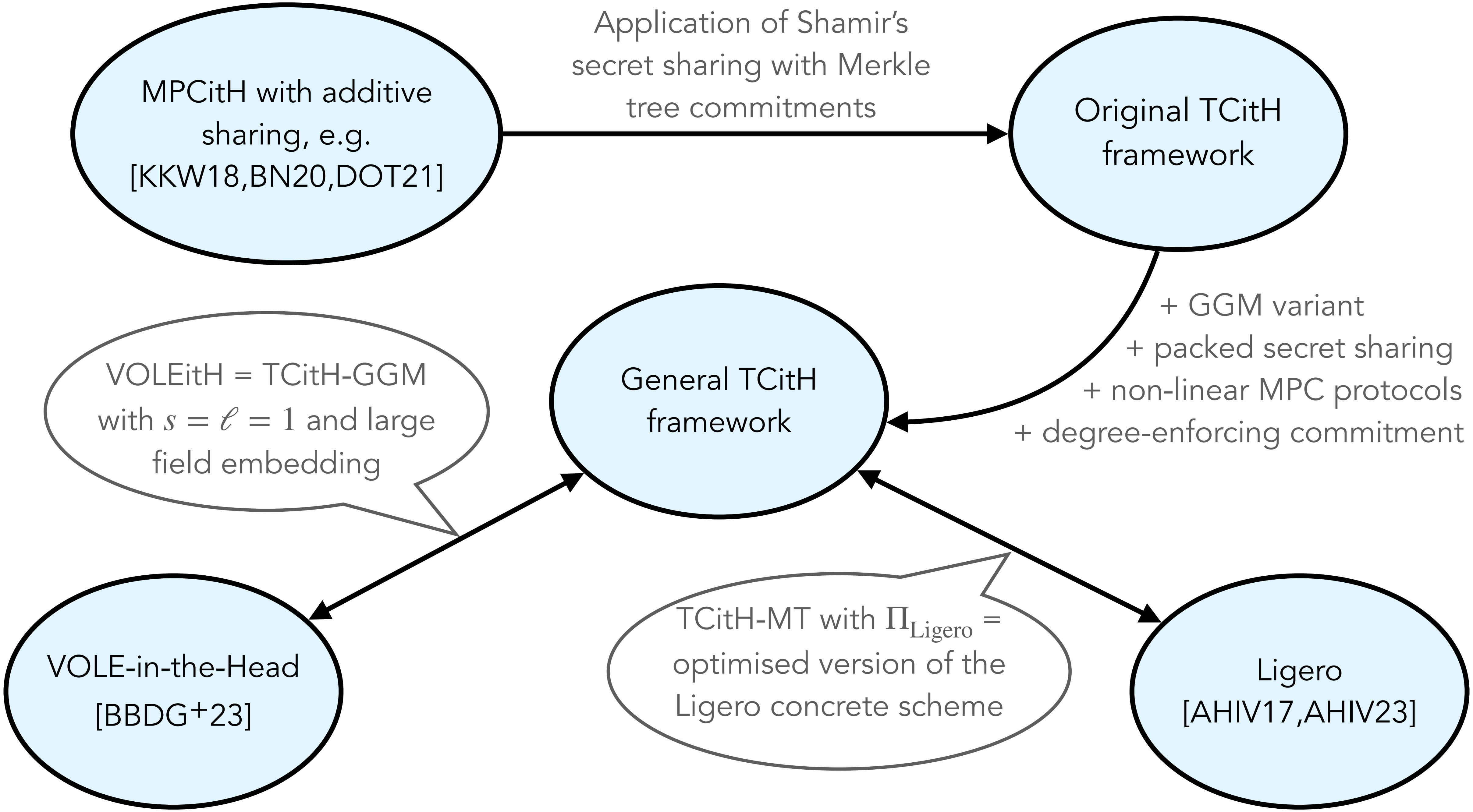
VOLEitH = TCitH-GGM with $s = \ell = 1$ and large field embedding

VOLE-in-the-Head [BBDG+23]

Connections to other proof systems



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VOLE-in-the-Head [BBDG$^+$23]

TCitH-MT with Π_{Ligero} = optimised version of the Ligero concrete scheme

Ligero [AHIV17,AHIV23]

Thank you!

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